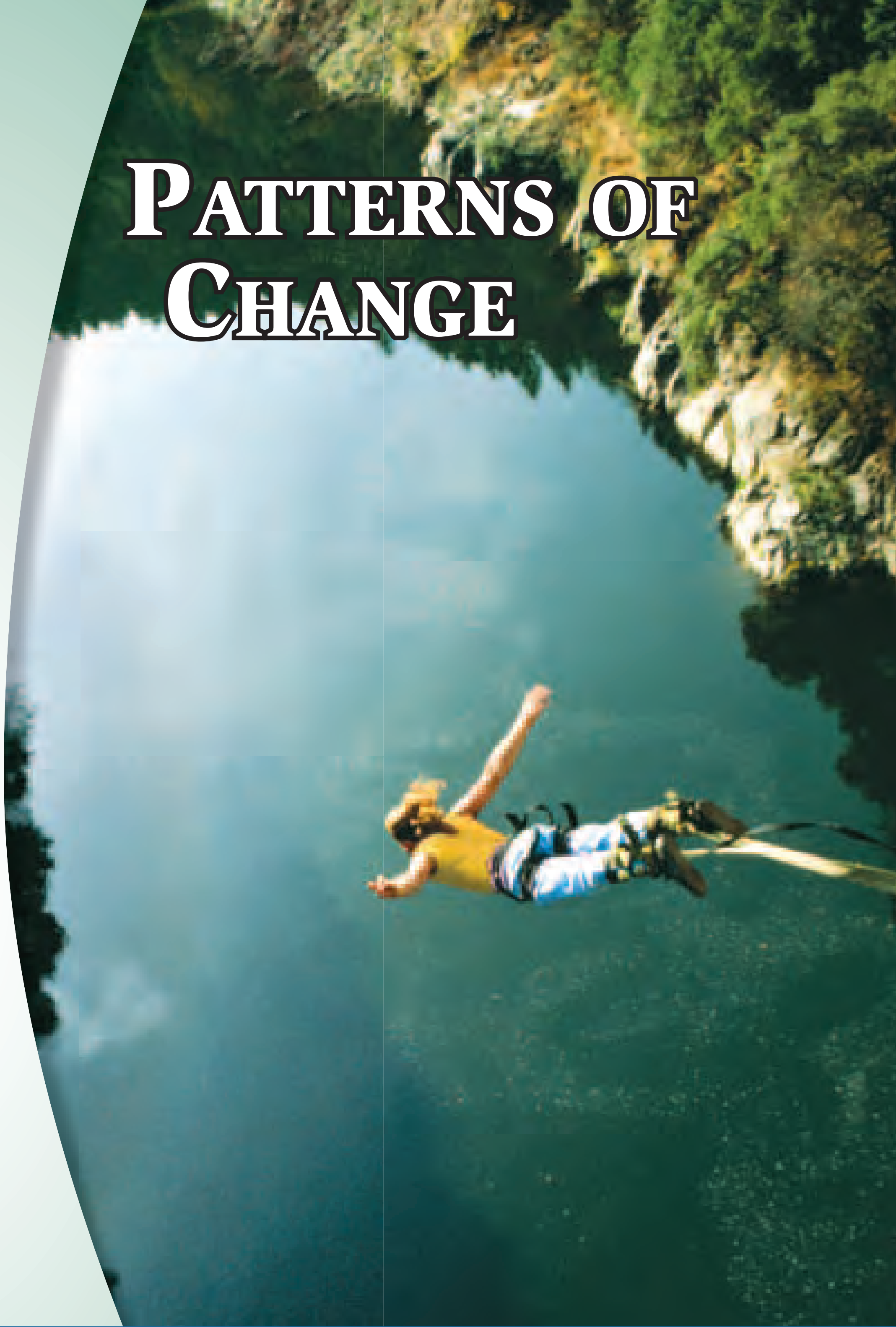
UNIT 1

Patterns of Change



Change is an important and often predictable aspect of the world in which we live. For example, in the thrill sport of bungee jumping, the stretch of the bungee cord is related to the weight of the jumper. A change in *jumper weight* causes change in *stretch of the cord*. Because bungee jumpers come in all sizes, it is important for jump operators to understand the connection between the key variables.

In this first unit of *Core-Plus Mathematics*, you will study ideas and reasoning methods of algebra that can be used to describe and predict patterns of change in quantitative variables. You will develop understanding and skill in use of algebra through work on problems in three lessons.

Lessons

1 *Cause and Effect*

Use tables, graphs, and algebraic rules to represent relationships between independent and dependent variables. Describe and predict the patterns of change in those cause-and-effect relationships.

2 *Change Over Time*

Use tables, graphs, and algebraic rules to describe, represent, and analyze patterns in variables that change with the passage of time. Use calculators and computer spreadsheets to study growth of populations and investments.

3 *Tools for Studying Patterns of Change*

Use calculator and computer tools to study relationships between variables that can be represented by algebraic rules. Explore connections between function rules and patterns of change in tables and graphs.

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LESSON 1

Cause and Effect

Popular sports like baseball, basketball, football, soccer, tennis, and golf have been played around the world for many years. But creative athletes are always looking for new thrills and challenges.

One new sport began when some young daredevils in New Zealand found a bridge over a deep river gorge and invented bungee jumping. They tied one end of a strong elastic cord to the bridge and the other end around their waists or feet. Then they jumped off the bridge and bounced up and down at the end of the cord to entertain tourists.

Soon word of this new sport got back to the United States. It wasn't long before some Americans tried bungee jumping on their own. Now amusement parks around the world have installed bungee jumps to attract customers who want a thrilling experience. Those parks had important planning to do before opening their bungee jumps for business.

As you can imagine, bungee jumping is risky, especially if the jump operator doesn't plan ahead carefully. If the apparatus isn't designed correctly, the consequences can be fatal. So, the first planning task is to make sure that the bungee apparatus is safe. Once the bungee jump is ready, it is time to consider business problems like setting prices to attract customers and maximize profit.

David R. Frazier

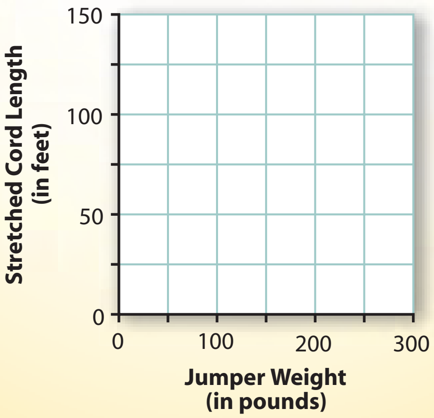
Think About This Situation

Suppose that operators of Five Star Amusement Park are considering installation of a bungee jump.

a How could they design and operate the bungee jump attraction so that people of different weights could have safe but exciting jumps?

b Suppose one test with a 50-pound jumper stretched a 60-foot bungee cord to a length of 70 feet. What patterns would you expect in a table or graph showing the stretched length of the 60-foot bungee cord for jumpers of different weights?

Jumper Weight (in pounds)	50	100	150	200	250	300
Stretched Cord Length (in feet)	70	?	?	?	?	?



c How could the Five Star Amusement Park find the price to charge each customer so that daily income from the bungee jump attraction is maximized?

d What other safety and business problems would Five Star Amusement Park have to consider in order to set up and operate the bungee attraction safely and profitably?

As you complete the investigations in this lesson, you will learn how data tables, graphs, and algebraic rules express relations among variables. You will also learn how they can help in solving problems and making decisions like those involved in design and operation of the Five Star bungee jump.

Investigation 1 Physics and Business at Five Star Amusement Park

The distance that a bungee jumper falls before bouncing back upward *depends on* the jumper's weight. In designing the bungee apparatus, it is essential to know how far the elastic cord will stretch for jumpers of different weights.

The number of customers attracted to an amusement park bungee jump depends on the price charged per jump. Market research by the park staff can help in setting a price that will lead to maximum income from the attraction.

As you work on the problems of this investigation, look for answers to these questions:

How is the stretch of a bungee cord related to the weight of the bungee jumper?

How are number of customers and income for a bungee jump related to price charged for a jump?

How can data tables, graphs, and rules relating variables be used to answer questions about such relationships between variables?

Bungee Physics In design of any amusement park attraction like a bungee jump, it makes sense to do some testing before opening to the public. You can get an idea about what real testing will show by experimenting with a model bungee apparatus made from rubber bands and small weights. The pattern relating jumper weight and cord stretch will be similar to that in a real jump.

When scientists or engineers tackle problems like design of a safe but exciting bungee jump, they often work in research teams. Different team members take responsibility for parts of the design-and-test process. That kind of team problem solving is also effective in work on classroom mathematical investigations.

As you collect and analyze data from a bungee simulation, you may find it helpful to work in groups of about four, with members taking specific roles like these:

Experimenters Perform the actual experiment and make measurements.

Recorders Record measurements taken and prepare reports.

Quality Controllers Observe the experiment and measurement techniques and recommend retests when there are doubts about accuracy of work.

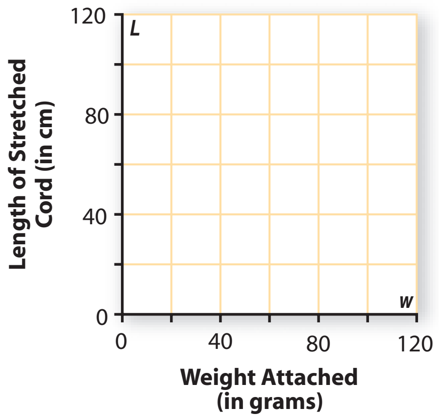
Different mathematical or experimental tasks require different role assignments. But, whatever the task, it is important to have confidence in your partners, to share ideas, and to help others.

Tim Fuller

1 Make a model bungee jump by attaching a weight to an elastic cord or to a chain of rubber bands.

a. Use your model to collect test data about bungee cord stretch for at least five weights. Record the data in a table and display it as a *scatterplot* on a graph.

Weight Attached (in grams)						
Length of Stretched Cord (in cm)						



b. Use the pattern in your experimental data to predict length of the stretched bungee cord for weights different from those already tested. Then test the accuracy of your predictions.

c. Compare your results to those of others doing the same experiment. What might explain any differences in results?

2 When a group of students in Iowa did the bungee jump experiment, they proposed an algebraic rule relating the length of the stretched bungee cord L (in centimeters) to the attached weight w (in grams). They said that the rule $L = 30 + 0.5w$ could be used to predict the stretched cord length for any reasonable weight.

a. Use the Iowa students' rule to make a table and a graph of sample (w, L) values for $w = 0$ to 120 in steps of 20 grams.

b. How, if at all, do the numbers 30 and 0.5 in the Iowa students' rule relate to the pattern of (w, L) values shown in the table and graph? What do they tell about the way the length of the cord changes as the attached weight changes?

c. Is the pattern of change in the rule-based (w, L) values in Part a different from the pattern of change in your experimental data? If so, what differences in experimental conditions might have caused the differences in results?

Tim Fuller

Bungee Business Designing the bungee jump apparatus is only part of the task in adding the attraction to Five Star Amusement Park. It is also important to set a *price per jump* that will make the operation profitable. When businesses face decisions like these, they get helpful information from market research. They ask people how much they would be willing to pay for a new product or service.

3 The Five Star marketing staff did a survey of park visitors to find out the *number of customers* that could be expected each day for the bungee jump at various possible *price per jump* values. Their survey produced data that they rounded off and presented in this table.

Market Survey Data

Price per Jump (in dollars)	0	5	10	15	20	25	30
Likely Number of Customers	50	45	40	35	30	25	20

a. Plot the (*price per jump, number of customers*) data on a coordinate graph. Then describe how the predicted *number of customers* changes as *price per jump* increases from \$0 to \$30.

b. The Five Star data processing department proposed the rule $N = 50 - p$ for the relationship between *number of customers* N and *price per jump* p. Does this rule represent the pattern in the market research data? Explain your reasoning.

4 The Five Star staff also wanted to know about *daily income* earned from the bungee jump attraction.

a. If the price per jump is set at \$5, the park can expect 45 bungee jump customers per day. In this case, what is the daily income?

b. Use the market survey data from Problem 3 to estimate the *daily income* earned by the bungee jump for prices from \$0 to \$30 in steps of \$5. Display the (*price per jump, daily income*) data in a table and in a graph. Then describe the pattern relating those variables.

c. What do the results of the Five Star market research survey and the income estimates suggest as the best price to charge for the bungee jump attraction? How is your answer supported by data in the table and the graph of (*price per jump, daily income*) values?

5 In situations where values of one variable depend on values of another, it is common to label one variable the **independent variable** and the other the **dependent variable.** Values of the dependent variable are a function of, or depend on, values of the independent variable. What choices of independent and dependent variables make sense in:

a. studying design of a bungee jump apparatus?

b. searching for the price per jump that will lead to maximum income?

Michael Newman/PhotoEdit

Summarize the Mathematics

To describe relationships among variables, it is often helpful to explain how one variable *is a function of* the other or how the value of one variable *depends on* the value of the other.

a How would you describe the way that:

i. the stretch of a bungee cord depends on the weight of the jumper?

ii. the number of customers for a bungee jump attraction depends on the price per customer?

iii. income from the jump depends on price per customer?

b What similarities and what differences do you see in the relationships of variables in the physics and business questions about bungee jumping at Five Star Amusement Park?

c In a problem situation involving two related variables, how do you decide which should be considered the independent variable? The dependent variable?

d What are the advantages and disadvantages of using tables, graphs, algebraic rules, or descriptions in words to express the way variables are related?

e In this investigation, you were asked to use patterns in data plots and algebraic rules to make predictions of bungee jump stretch, numbers of customers, and income. How much confidence or concern would you have about the accuracy of those predictions?

Be prepared to share your thinking with the whole class.

Check Your Understanding

The design staff at Five Star Amusement Park had another idea—selling raffle tickets for chances to win prizes. The prize-winning tickets would be drawn at random each day.

a. Suppose that a market research study produced the following estimates of raffle ticket sales at various prices.

Price per Ticket (in dollars)	1	2	3	4	5	10	15
Number of Tickets Sold	900	850	800	750	700	450	200

i. Plot the (*price per ticket, number of tickets sold*) estimates on a graph. Because *price per ticket* is the independent variable in this situation, its values are used as x-coordinates of the graph. Because *number of tickets sold* is the dependent variable, its values are used as the y-coordinates of the graph.

ii. Describe the pattern relating values of those variables and the way that the relationship is shown in the table and the graph.

iii. Does the rule $N = 950 - 50p$ produce the same pairs of (*price per ticket p, number of tickets sold N*) values as the market research study?

b. Use the data in Part a relating *price per ticket* to *number of tickets sold* to estimate the *income* from raffle ticket sales at each of the proposed ticket prices.

i. Record those *income* estimates in a table and plot the (*price per ticket, income*) estimates on a graph.

ii. Describe the relationship between raffle ticket price and income from ticket sales. Explain how the relationship is shown in the table and the graph of (*price per ticket, income*) estimates.

iii. What do your results in parts i and ii suggest about the ticket price that will lead to maximum income from raffle ticket sales? How is your answer shown in the table and graph of part i?

Investigation 2 Taking Chances

Students at Banneker High School hold an annual *Take a Chance* carnival to raise funds for special class projects. The planning committee is often puzzled about ways to predict profit from games of chance.

In one popular game, a fair die is rolled to find out whether you win a prize. Rules of the game are:

- You win a $4 prize if the top face of the die is a 4.
- You donate $1 to the school special project fund if the top face of the die is 1, 2, 3, 5, or 6.

As you work on the problems of this investigation, look for answers to these questions:

What is the pattern of change relating profit to number of players in the die-tossing game?

How is that pattern of change illustrated in tables and graphs of data from plays of the game?

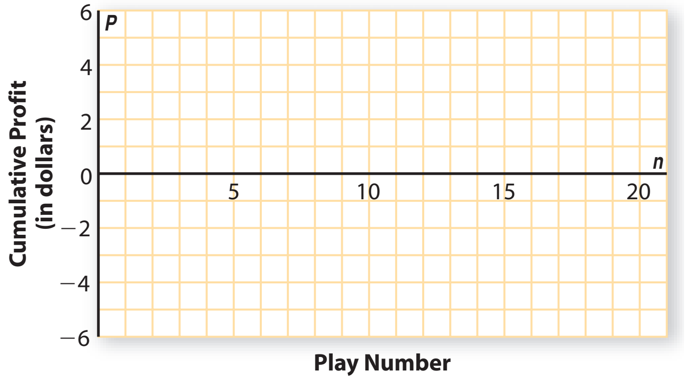
How is the pattern of change in profit similar to and different from the patterns of change in bungee jump cord length and number of bungee jump customers?

1 Use a fair die to play the die-tossing game at least 20 times. Record your results in a table like this:

Play Number	1	2	3	4	5	6	7	...
Outcome ($ won or lost for school)								...
Cumulative Profit ($ won or lost by school)								...

Index Stock

2 Plot the data from your test of the game on a graph that shows how *cumulative profit* for the school changes as the *number of plays* increases. Since the school can lose money on this game, you will probably need a graph (like the one below) showing points below the horizontal axis. Connecting the plotted points will probably make patterns of change in fund-raiser profit clearer. Use the graph to answer the questions that follow.



a. Describe the pattern of change in profit or loss for the school as clearly and precisely as you can. Explain how the pattern is shown in the table and the graph.

b. See if you can express the pattern as a rule relating *cumulative profit* P to *number of plays* n.

3 Combine your results from the die-tossing experiment with those of other students to produce a table showing results of many more plays. If each student or group contributes cumulative results for 20 plays, you could build a table like this:

Number of Plays	20	40	60	80	100	120	140	160
Cumulative Profit (in $)								

Plot the resulting (*number of plays, cumulative profit*) data.

a. How is the pattern in this experiment with many plays similar to or different from the patterns of your experiment with fewer plays?

b. See if you can express the pattern as a rule relating *cumulative profit* P to *number of plays* n.

4 Suppose that the game operators change the prize payoff from $4 to $6.

a. What similarities and differences would you expect in the way *cumulative profit* for the school changes as the *number of plays* increases in the new game compared to the original game? How will those patterns appear in a data table and a graph of results?

b. Repeat the die-tossing experiment to test profit prospects for the fund-raiser with the new payoff scheme. Try to explain differences between what you predicted would happen in Part a and what actually did happen.

5 What payoff amounts (for winning and losing) might make this a fair game—that is, a game in which profit for the school is expected to be zero?

Summarize the Mathematics

In this investigation, you explored patterns of change for a variable with outcomes subject to the laws of probability. You probably discovered in the die-tossing game that *cumulative profit* is related somewhat predictably to the *number of plays* of the game.

a After many plays of the two games with payoffs of $4 or $6, who seemed to come out ahead in the long run—the players or the school fund-raiser? Why do you think those results occurred?

b How is the pattern of change in *cumulative profit* for the school fund-raiser similar to, or different from, patterns you discovered in the investigation of bungee physics and business?

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

Suppose that another game at the *Take a Chance* carnival has these rules:

Three coins—a nickel, a dime, and a quarter—are flipped.

If all three turn up heads or all three turn up tails, the player wins a $5 prize.

For any other result, the player has to contribute $2 to the school fund.

The school fund-raiser is most likely to win $2 on any individual play of the game, but there is also a risk of losing $5 to some players. The challenge is to predict change in fund-raiser profit as more and more customers play this game.

a. If you keep a tally of your *cumulative profit* (or loss) for many plays of this game:

i. What pattern would you expect to find in your cumulative profit as the number of plays increases?

ii. How would the pattern you described in part i appear in a graph of the recorded (*play number, cumulative profit*) data?

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b. Use three coins to play the game at least 20 times. In a table, record the results of each play and the cumulative profit (or loss) after each play. Make a plot of your (*play number, cumulative profit*) data and describe the pattern shown by that graph.

c. How are the results of your actual plays similar to what you predicted in Part a? If there are differences, how can they be explained?

Investigation 3 Trying to Get Rich Quick

Relationships between independent and dependent variables occur in a wide variety of problem situations. Tables, graphs, and algebraic rules are informative ways to express those relationships. The problems of this investigation illustrate two other common patterns of change. As you work on the next problems—about NASCAR racing and pay-for-work schemes—look for answers to these questions:

Why are the relationships involved in these problems called nonlinear patterns of change?

How do the dependent variables change as the independent variables increase?

NASCAR Racing Automobile racing is one of the most popular spectator sports in the United States. One of the most important races is the NASCAR Daytona 500, a 500-mile race for cars similar to those driven every day on American streets and highways. The prize for the winner is over $1 million. Winners also get lots of advertising endorsement income.

1 The average speed and time of the Daytona 500 winner varies from year to year.

a. In 1960, Junior Johnson won with an average speed of 125 miles per hour. The next year Marvin Panch won with an average speed of 150 miles per hour. What was the difference in race time between 1960 and 1961 (in hours)?

b. In 1997, Jeff Gordon won with an average speed of 148 miles per hour. The next year the winner was Dale Earnhardt with an average speed of 173 miles per hour. What was the difference in race time between 1997 and 1998 (in hours)? (Source: www.nascar.com/races/)

2 Complete a table like that shown here to display sample pairs of (*average speed, race time*) values for completion of a 500-mile race.

Average Speed (in mph)	50	75	100	125	150	175	200
Race Time (in hours)							

AP/Wide World Photos

a. Plot the sample (*average speed, race time*) data on a graph. Describe the relationship between those two variables.

b. Write a symbolic rule that shows how to calculate *race time t* as a function of *average speed s* in the Daytona 500 race. Show with specific examples that your rule produces correct race time for given average speed.

3 In the 1960–61 and 1997–98 comparisons of winning speed and time for the Daytona 500 race, the differences in average speed are both 25 miles per hour. The time differences are not the same. At first, this might seem like a surprising result.

How is the fact that equal changes in average speed don't imply equal changes in race time illustrated in the shape of the graph of sample (*average speed, race time*) data?

4 How are the table, graph, and algebraic rule relating *average speed* and *race time* similar to or different from those you have seen in work on earlier problems?

Part-Time Work ... Big-Time Dollars When Devon and Kevin went looking for part-time work to earn spending money, their first stop was at the Fresh Fare Market. They asked the manager if they could work helping customers carry groceries to their cars. When the manager asked how much they wanted to earn, Devon and Kevin proposed $2 per hour plus tips from customers.

The Fresh Fare Market manager proposed a different deal, to encourage Devon and Kevin to work more than a few hours each week. The manager's weekly plan would pay each of them $0.10 for the first hour of work, $0.20 for the second hour, $0.40 for the third hour, $0.80 for the fourth hour, and so on.

5 Which pay plan do you think would be best for Devon and Kevin to choose? To provide evidence supporting your ideas, complete a table showing the earnings (without tips) for each student from each plan for work hours from 1 to 10. Plot graphs showing the patterns of growth in earnings for the two plans.

Hours Worked in a Week	1	2	3	4	5	6	7	8	9	10
Earnings in $ Plan 1	2	4	...							
Earnings in $ Plan 2	0.10	0.30	...							

Based on the pattern of earnings, which of the two pay plans would you recommend to Devon and Kevin?

Kimberly White/Reuters/CORBIS

6 Would you change your choice of pay plan if the manager's offer was:

a. Plan 3: Only \$0.05 for the first hour of work, \$0.10 for the second hour, \$0.20 for the third hour, \$0.40 for the fourth hour, and so on? Why or why not?

b. Plan 4: Only \$0.01 for the first hour of work, but \$0.03 for the second hour, \$0.09 for the third, \$0.27 for the fourth, and so on? Why or why not?

Summarize the Mathematics

The patterns relating *race time* to *average speed* for the Daytona 500 and *earnings* to *hours worked* in Plan 2 at Fresh Fare Market are examples of nonlinear relationships.

a What is it about those relationships that makes the term "nonlinear" appropriate?

b You found patterns showing how to calculate *race time* from *average speed* and *total pay* from *hours worked*. How would your confidence about the accuracy of those calculations compare to that for calculations in the bungee jump and fair game problems?

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

Use these problems to test your skill in analyzing nonlinear relationships like those in the NASCAR and Fresh Fare Market problems.

a. The Iditarod Trail Sled Dog Race goes 1,100 miles from Anchorage to Nome, Alaska, in March of each year. The winner usually takes about 10 days to complete the race.

i. What is a typical average speed (in miles per day) for Iditarod winners?

ii. Make a table and sketch a graph showing how *average speed* for the Iditarod race depends on *race time*. Use times ranging from 2 (not really possible for this race) to 20 days in steps of 2 days.

iii. What rule shows how to calculate *average speed s* for any Iditarod *race time t*?

iv. Compare the table, graph, and rule showing Iditarod *average speed* as a function of *race time* to that showing Daytona 500 *race time* as a function of *average speed* in Problem 2 of this investigation. Explain how relationships in the two situations are similar and how they are different.

b. Ethan and Anna tried to get a monthly allowance of spending money from their parents. They said, "You only have to pay us 1 penny for the first day of the month, 2 pennies for the second day of the month, 4 pennies for the third day, and so on." According to Ethan and Anna's idea, how much would the parents have to pay on days 10, 20, and 30 of each month?

Kevin Horan/Getty Images

On Your Own

Applications

These tasks provide opportunities for you to use and strengthen your understanding of the ideas you have learned in the lesson.

1 The table below gives data from tests of a full-size bungee jump.

Jumper Weight (in pounds)	100	125	150	175	200
Stretched Cord Length (in feet)	50	55	60	65	70

a. Which variable does it make sense to consider independent and which dependent?

b. Plot the given data on a coordinate graph.

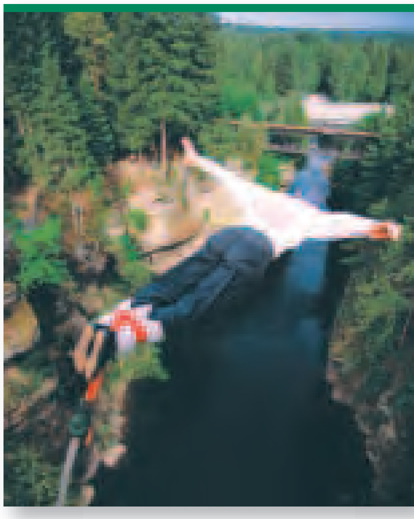
c. Use the pattern in the table or the graph to estimate the stretched cord length for jumpers who weigh:

i. 85 pounds **ii.** 135 pounds **iii.** 225 pounds

d. Would it make sense to connect the points on your data plot? Explain your reasoning.

e. Describe the overall pattern relating *stretched cord length L* to *jumper weight w.*

f. The technician who did the tests suggested that the pattern could be summarized with a symbolic rule $L = 30 + 0.2w$. Does that rule give estimates of stretched cord length that match the experimental data? Explain.



2 To help in estimating the number of customers for an amusement park bungee jump, the operators hired a market research group to visit several similar parks that had bungee jumps. They recorded the number of customers on a weekend day. Since the parks charged different prices for their jumps, the collected data looked like this:

Price per Jump (in dollars)	15	20	25	28	30
Number of Customers	25	22	18	15	14

a. In this situation, which variable makes sense as the independent variable and which as the dependent variable?

b. Plot these data on a coordinate graph.

Getty Images

c. Does it make sense to connect the points on your data plot? Explain your reasoning.

d. Use the pattern in the table or the graph to estimate the *number of customers* if the *price per jump* is:

i. \$18 **ii.** \$23 **iii.** \$35

e. Describe the overall pattern of change relating *price per jump* to *number of customers.*

f. The market research staff suggested that the pattern could be summarized with a rule $n = 35 - 0.7p$. Does that rule produce estimates of number of customers n at various prices p like those in the survey data?

3 Use the data in Applications Task 2 to study the relationship between price per bungee jump and income from one day's operation at the five parks that were visited.

a. Complete a table showing sample (*price per jump, daily income*) values.

Price per Jump (in dollars)	15	20	25	28	30
Daily Income (in dollars)					

b. In this situation, what choice of independent and dependent variables makes most sense?

c. Plot the data relating *price per jump* and *daily income* on a coordinate graph.

d. Would it make sense to connect the points on the graph? Explain your reasoning.

e. Describe the overall pattern in the relationship between *price per jump* and *daily income.*

f. Use the data table and graph pattern to estimate the *price per jump* that seems likely to yield maximum *daily income.*

4 Suppose that you go to a school carnival night and play a game in which two fair coins are tossed to find out whether you win a prize. The game has these rules:

- Two heads or two tails showing—you win \$1.
- One head and one tail showing—you lose \$1.

a. If you keep score for yourself in 20 plays of this game:

i. What pattern would you expect in your cumulative score as the plays occur?

ii. How would the pattern you described in part i appear in a graph of (*play number, cumulative score*) data?

b. Use two coins to play the game 20 times. Record the results of each play and the cumulative score after each play in a table. Make a scatterplot of your (*play number, cumulative score*) data and describe the pattern shown by that graph.

c. How are the results of your actual plays similar to what you predicted in Part a? How are they different?

5 The postage cost for U.S. first-class mail is related to the weight of the letter or package being shipped. The following table gives the regulations in 2006 for relatively small letters or packages.

Weight (in ounces)	up to 1	up to 2	up to 3	up to 4	up to 5
Postage Cost (in dollars)	0.39	0.63	0.87	1.11	1.35

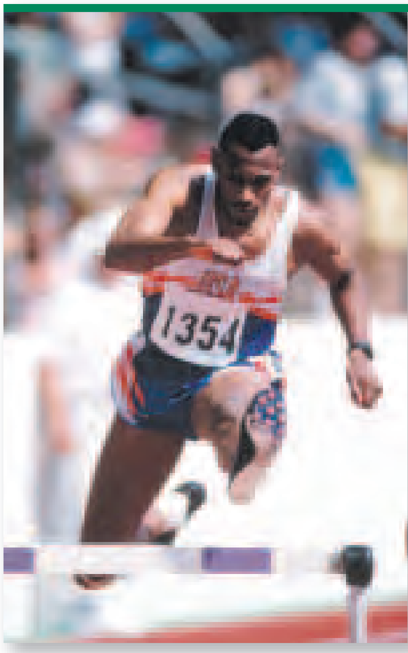
a. Make a coordinate graph showing (*weight, postage cost*) values for letters or packages weighing 1, 2, 3, 4, and 5 ounces.

b. What postage costs would you expect for letters or small packages sent by first-class mail, if those items weighed:

i. 1.5 ounces **ii.** 4.25 ounces **iii.** 7 ounces

c. Add the (*weight, postage cost*) values from Part b to your graph. How should the points on your graph be connected (if at all)?

6 The Olympic record for the men's 400-meter hurdle race is 46.78 seconds. It was set by Kevin Young in 1992. His average running speed was $400 \div 46.78 \approx 8.55$ meters per second.



a. Make a table and a graph showing how 400-meter *race time* changes as *average speed* increases from 2 meters per second to 10 meters per second in steps of 1 meter per second.

b. Describe the pattern of change shown in your table and graph.

c. Write a rule showing how to calculate *race time t* for any *average speed s*.

d. Which change in *average speed* will reduce the *race time* most: an increase from 2 to 4 meters per second or an increase from 8 to 10 meters per second? Explain how your answer is illustrated in the shape of your graph.

7 The Olympic record in the women's 100-meter freestyle swim race is 53.52 seconds. It was set by Australian Jodie Henry in 2004. She swam at an average speed of $100 \div 53.52 \approx 1.87$ meters per second.

a. Make a table and a graph showing the way *average speed* for the 100-meter race changes as *time* increases from 40 seconds to 120 seconds (2 minutes) in steps of 10 seconds.

S. Carmona/CORBIS

b. Describe the pattern of change shown in your table and graph.

c. Write a rule showing how to calculate *average speed s* for any *race time t*.

d. Which change in *race time* will cause the greatest change in *average speed*: an increase from 50 to 60 seconds or an increase from 110 to 120 seconds? Explain how your answer is illustrated in the shape of your graph.

8 The Water World Amusement Park has a huge swimming pool with a wave machine that makes you feel like you are swimming in an ocean. Unfortunately, the pool is uncovered and unheated, so the temperature forecast for a day affects the number of people who come to Water World.

On a summer day when the forecast called for a high temperature of 90°F, about 3,000 people visited the park. On another day, when the forecast called for a high temperature of 70°F, only 250 people came for the ocean-wave swimming.

a. Complete this table of (*temperature forecast, number of swimmers*) data in a way that you think shows the likely pattern relating *temperature forecast* to *number of swimmers.*

Temperature Forecast (in °F)	70	75	80	85	90	95
Number of Swimmers	250				3,000	

b. Graph the data in Part a. Then draw a line or curve that seems to match the pattern in your data points and could be used to predict *number of swimmers* at other temperatures.

c. Describe the pattern of change in *number of swimmers* as *temperature forecast* increases and explain how much confidence you would have in using that pattern to predict *number of swimmers* on any particular day.

d. Use your table and/or graph to estimate the number of swimmers for temperatures of:

i. 77°F **ii.** 83°F **iii.** 98°F

e. Suppose that Water World charges $15 for admission. Use this information and your estimates for number of swimmers at various forecast temperatures to make a table and graph showing the relationship between *forecast high temperature* and *park income.*

f. Use the information from Part e to estimate the park income when the high temperature is forecast to be:

i. 87°F **ii.** 92°F

g. Why would you have limited confidence in using the data patterns of Parts a and e to predict park income when the forecast high temperature is 40°F or 110°F?

Sergio Dorantes/CORBIS

Connections

These tasks will help you to build links between mathematical topics you have studied in the lesson and to connect those topics with other mathematics that you know.

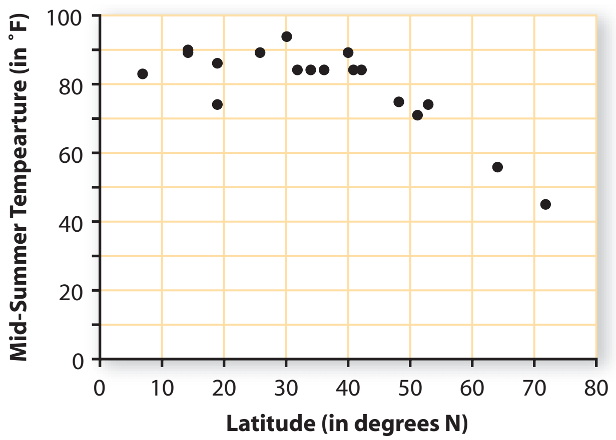
9 The table below shows latitude of some major northern hemisphere cities and the average high temperatures in those cities in mid-summer and in mid-winter. Use that data and the scatterplots on page 19 to answer Parts a–c about the relationship between latitude and typical temperatures.

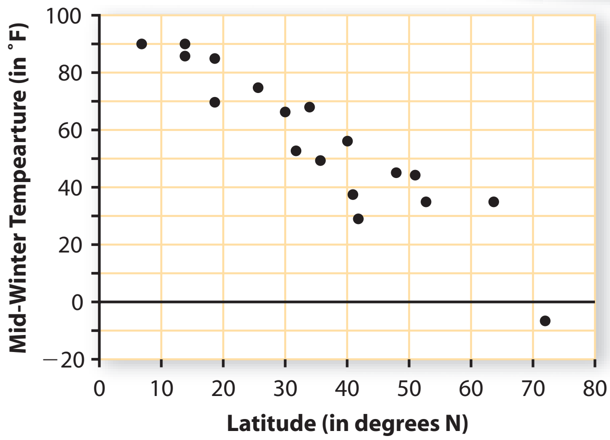
a. Does the pattern of points relating *mid-summer average high temperature* to *geographic latitude* suggest a close relationship between those variables? Explain your reasons for saying yes or no.

b. Does the pattern of points relating *mid-winter average high temperature* to *geographic latitude* suggest a close relationship between those variables? Explain your reasons for saying yes or no.

c. What factors other than latitude might influence summer and winter temperatures?

City	Latitude Degrees N	Mid-Summer Temperature °F	Mid-Winter Temperature °F
Athens, Greece	40	89	56
Bangkok, Thailand	14	90	90
Barrow, Alaska	72	45	–7
Berlin, Germany	53	74	35
Bombay, India	19	86	85
Cairo, Egypt	30	94	66
Chicago, Illinois	42	84	29
Jerusalem, Israel	32	84	53
Lagos, Nigeria	7	83	90
London, England	51	71	44
Los Angeles, California	34	84	68
Mexico City, Mexico	19	74	70
Miami, Florida	26	89	75
Manila, Philippines	14	89	86
New York City, New York	41	84	37
Reykjavik, Iceland	64	56	35
Seattle, Washington	48	75	45
Tokyo, Japan	36	84	49





10 Random events such as the outcomes of flipping a fair coin often have predictable patterns.

a. What is the probability of flipping a coin once and getting a head?

b. What is the probability of flipping a coin two times and getting two heads?

c. What is the probability of flipping a coin three times and getting three heads?

d. What is the probability of flipping a coin four times and getting four heads?

e. How would you describe the pattern in the probabilities of getting all heads as the number of coin tosses increases?

11 Jamal's average on history quizzes changed throughout the first quarter.

a. After the first two quizzes, his average was 7, but he earned a 9 on the third quiz. What was his average for the first three quizzes?

b. After the first eight quizzes, his average had slipped again to 7, but he earned 9 on the ninth quiz. What was his average for all nine quizzes of the quarter?

c. Why did Jamal's 9 on the third quiz improve his overall average more than his 9 on the ninth quiz?

12 When the value of a quantity changes, there are several standard ways to describe *how much* it has changed. For example, if a boy who is 60 inches tall at the start of grade 8 grows to 66 inches twelve months later, we could say his height has increased:

- by 6 inches (the *difference* between original and new height)
- by 10% (the *relative* or *percent change* in his height)
- by 0.5 inches per month (an *average rate of change*)

Express each of the following quantitative changes in three ways similar to those above:

a. The enrollment of Wayzata High School increased from 1,000 to 1,250 in the five-year period from 1998 to 2003.

b. The balance in a student's bank savings account increased from \$150 to \$225 while she worked during the three-month summer break from school.

c. The supply of soft drinks in a school vending machine decreased from 200 to 140 during the 8 hours of one school day.

d. From the start of practice in March until the end of the track season in June, Mike's time in the 800-meter race decreased from 2 minutes 30 seconds to 2 minutes.

13 The related variables you studied in Investigations 1–3 and in Applications Tasks 1–8 are only a few of the many situations in which it helps to understand the pattern relating two or more variables.

a. Write a sentence in the form "__________ depends on __________" or "__________ is a function of __________" that describes a situation with which you are familiar.

b. For the situation you described in Part a:

i. Explain how change in one variable relates to or causes change in the other.

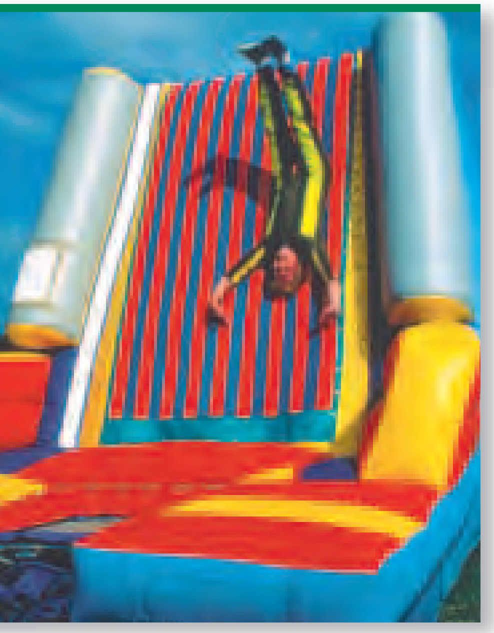
ii. Make a table showing at least 5 sample pairs of values that you would expect for the related variables.

iii. Plot a graph of the sample data in part ii and connect the points in a way that makes sense (if at all).

Reflections

These tasks provide opportunities for you to re-examine your thinking about ideas in the lesson.

14 Experimentation with one bungee cord suggested that the rule $L = 30 + 0.2w$ would be a good predictor of the stretched cord length as a function of jumper weight. The operators of the bungee jump decided to adjust the jump-off point for each jumper to the height L calculated from the rule. What reasons can you think of to question that plan?



15 The student government at Banneker High School decided to set up a Velcro® jump (pictured at the left) as a fund-raiser for a school trip. They did a survey to see how many students would try the Velcro jump at various prices.

The data were as follows:

Price per Jump (in dollars)	0.50	1.00	2.00	3.00	5.00
Expected Number of Jumps	95	80	65	45	15

When several groups of Banneker mathematics students were asked to study the survey data about profit prospects of the rented Velcro jump, they produced different kinds of reports.

How would you rate each of the following reports, on a scale of 5 (excellent) to 0 (poor)? Explain why you gave each report the rating you did.

Alamy Images

Report a: ***Making Money for Banneker***

The survey shows that a price of $0.50 will lead to the most customers, so that will bring in the biggest profit.

Report b: ***Sticking it to the Velcro Customers***

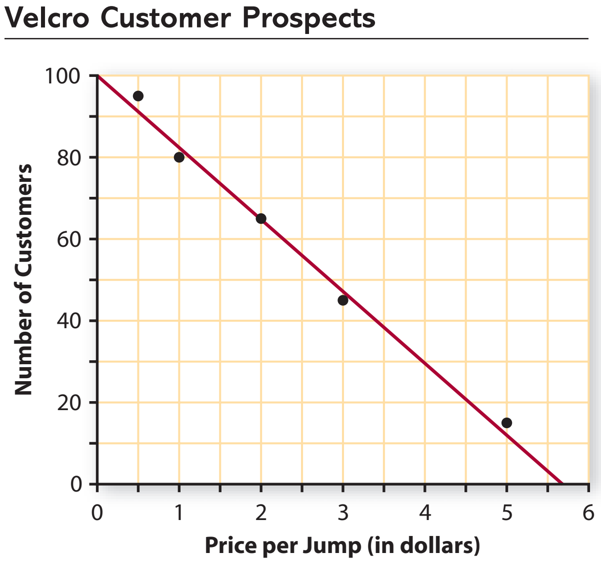
The survey shows that the more you charge, the fewer customers you will have. If you multiply each price by the expected number of customers, you get a prediction of the income from the Velcro jump.

When we did that, we found that a price of $3.00 leads to the greatest income, so that is what should be charged. If you want to let the most kids have fun, you should charge only $0.50. If the operators don't want to work very hard, they should charge $10, because then no one will want to pay to jump.

Report c: ***Velcro Profit Prospects***

Data from our market survey suggest a pattern in which the number of customers will decrease as the price increases. Each increase of $1 in the price will lead to a decrease of 15–20 customers. This pattern is shown in a plot of the survey data.

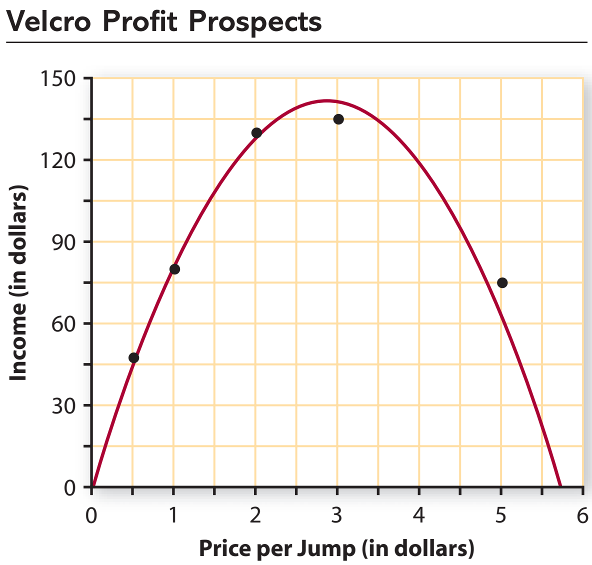
The trend in the data is matched well by the line drawn on the graph that follows. That line also helps in predicting the number of customers for prices not included in the survey.



To see how the amount of money earned by the Velcro game would be related to the price per jump, we added another row to the table, showing income. For example, 95 customers at $0.50 per jump will bring income of $47.50.

Price per Jump (in dollars)	0.50	1.00	2.00	3.00	5.00
Expected Number of Jumps	95	80	65	45	15
Expected Income (in dollars)	47.50	80.00	130.00	135.00	75.00

A graph of the *price per jump* and *income* data is shown at the right. It suggests that a price between \$2 and \$3 per jump will lead to the greatest income. Since the rental charge is a fixed dollar amount, greatest income means greatest profit.



16 If you were asked to look for a pattern relating the values of two variables in a problem, would you prefer to have:

- a table of (x, y) data,
- a plot of points with coordinates (x, y), or
- a symbolic rule showing how values of y could be calculated from values of x?

Explain the reasons for your choice.

17 When there appears to be a relationship between values of two variables, how do you decide which should be considered the *independent variable* and which should be considered the *dependent variable*?

Extensions

These tasks provide opportunities for you to explore further or more deeply the mathematics you are learning.

18 Suppose that for a fund-raising event, your school can rent a climbing wall for \$275. Complete the following tasks to help find the likely profit from using the climbing wall at the event.

AP/Wide World Photos

a. Do a survey of your class to find out how many customers you might expect for various possible prices. Then use your data to estimate the number of students from your whole school who would try the climbing wall at various possible prices.

Climb Price (in dollars)	1	2	3	4	5	6	7	8	9	10
Number of Customers										

b. Plot a graph of the survey data and explain how it shows the pattern of change relating *number of customers* to *climb price*. Be sure to explain which number it makes sense to consider the independent variable and which the dependent variable in this situation.

c. Display the data relating *number of customers* to *climb price* in a table and a graph. Then use the pattern in the data to estimate the *income* that would be earned at various possible prices.

d. What do you recommend as the price that will maximize *profit* from the climbing wall rental? Explain how your decision is based on patterns in the data tables and graphs you've displayed.

19 One of the most important principles of physics is at work when two kids play on a teeter-totter. You probably know that for two weights on opposite sides of the *fulcrum* to balance, those weights need to be placed at just the right distances from the fulcrum.

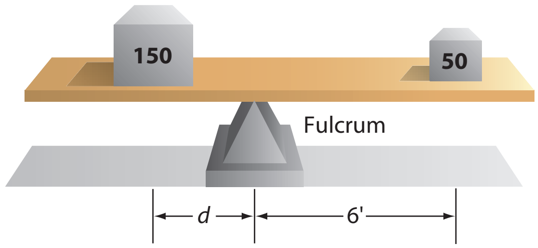
a. Suppose that a 50-pound weight is placed at one end of a teeter-totter, 6 feet from the fulcrum. How far from the fulcrum should a person sit to balance the 50-pound weight if the person weighs:

i. 50 pounds

ii. 100 pounds

iii. 150 pounds

(If you are unsure of the physical relationship required to make a balance, do some experiments with a meter stick as the teeter-totter and stacks of pennies as the weights.)



b. Sketch a graph showing the distance from the fulcrum required for various weights to balance a 50-pound weight that has been placed on the opposite side, 6 feet from the fulcrum. Describe the pattern relating distance from the fulcrum to the counter-balancing weight.

c. What rule relates distance from the fulcrum d (in feet) to weight w (in pounds) when the weight balances a 50-pound weight on the opposite side and 6 feet from the fulcrum?

Ole Graf/Zefa/CORBIS

20 In many problems, it is helpful to express the relationship between dependent and independent variables with a symbolic rule that shows how values of one variable can be calculated from the values of the other.

a. If number of customers n at a bungee jump is related to price per jump p by the rule $n = 50 - p$, what rule shows how to calculate income I from values of n and p? What rule shows how to calculate values of I from the value of p alone?

b. What rule shows how to calculate Ethan and Anna's allowance on day n of a month if they receive 1 penny on day one, 2 pennies on day two, 4 pennies on day three, 8 pennies on day four, and so on?

c. Devon and Kevin were offered a pay scheme for work at Fresh Fare Market that would earn each of them \$0.10 for the first hour in a week, \$0.20 for the second hour, \$0.40 for the third hour, \$0.80 for the fourth hour, and so on. What rule shows how to calculate their pay for the nth hour in a week?

Review

These tasks provide opportunities for you to review previously learned mathematics and to refine your skills in using that mathematics.

21 Suppose a fair die is rolled.

a. What is the probability that the top face is a 6?

b. What is the probability that the top face is a 3?

c. What is the probability that the top face is an even number?

d. What is the probability that the top face is *not* a 6?

22 Micah and Keisha are renting a boat. The charge for the boat is \$25 for the first hour and \$12 for every hour (or portion of an hour) after the first.

a. How much will it cost if they rent the boat at 1:00 P.M. and return it at 3:50 P.M.?

b. They have been saving money all summer and have \$80. What is the maximum amount of time that they can keep the boat?

23 The speed at which you travel, the length of time you travel, and the distance you travel are related in predictable ways. In particular, *speed* • *time* = *distance*. Use this relationship to help you answer the following questions.

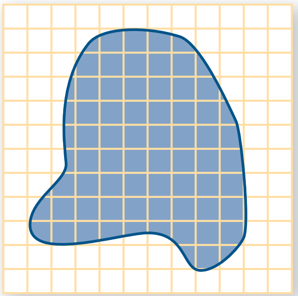
a. Dave rides his bike for 2 hours with an average speed of 8.6 miles per hour. How far does he travel?

b. Kristen lives 4 miles from her friend's house. It is 2:30 P.M. and she needs to meet her friend at 3:00 P.M. How fast must she ride her bike in order to get to her friend's house on time?

c. Jessie leaves home at 7:30 A.M. and rides his bike to school at a speed of 9 miles per hour. If his school is 3 miles from his house, what time will he get to school?

Index Stock

24 Consider this scale drawing of Mongoose Lake. Using the given scale, estimate the perimeter to the nearest 10 meters and the area to the nearest 100 square meters.



Mongoose Lake
Scale: ⊢⊣ = 10 meters

25 Convert each of these percents into equivalent decimals.

a. 75%

b. 5.4%

c. 0.8%

d. 0.93%

26 The table below gives some measurements associated with four different rectangles. Use the relationships between the lengths of the sides of a rectangle and the area and perimeter of a rectangle to complete the table.

Length (in cm)	**Width** (in cm)	**Perimeter** (in cm)	**Area** (in cm^2)
25	10	?	?
15	?	42	?
?	25	?	150
?	?	28	40

27 Convert each of these decimals into equivalent percents.

a. 0.8

b. 0.25

c. 2.45

d. 0.075

28 Suppose that a student has $150 in a bank savings account at the start of the school year. Calculate the change in that savings account during the following year in case it

a. earns 5% interest over that year.

b. grows from monthly deposits of $10 throughout the year.

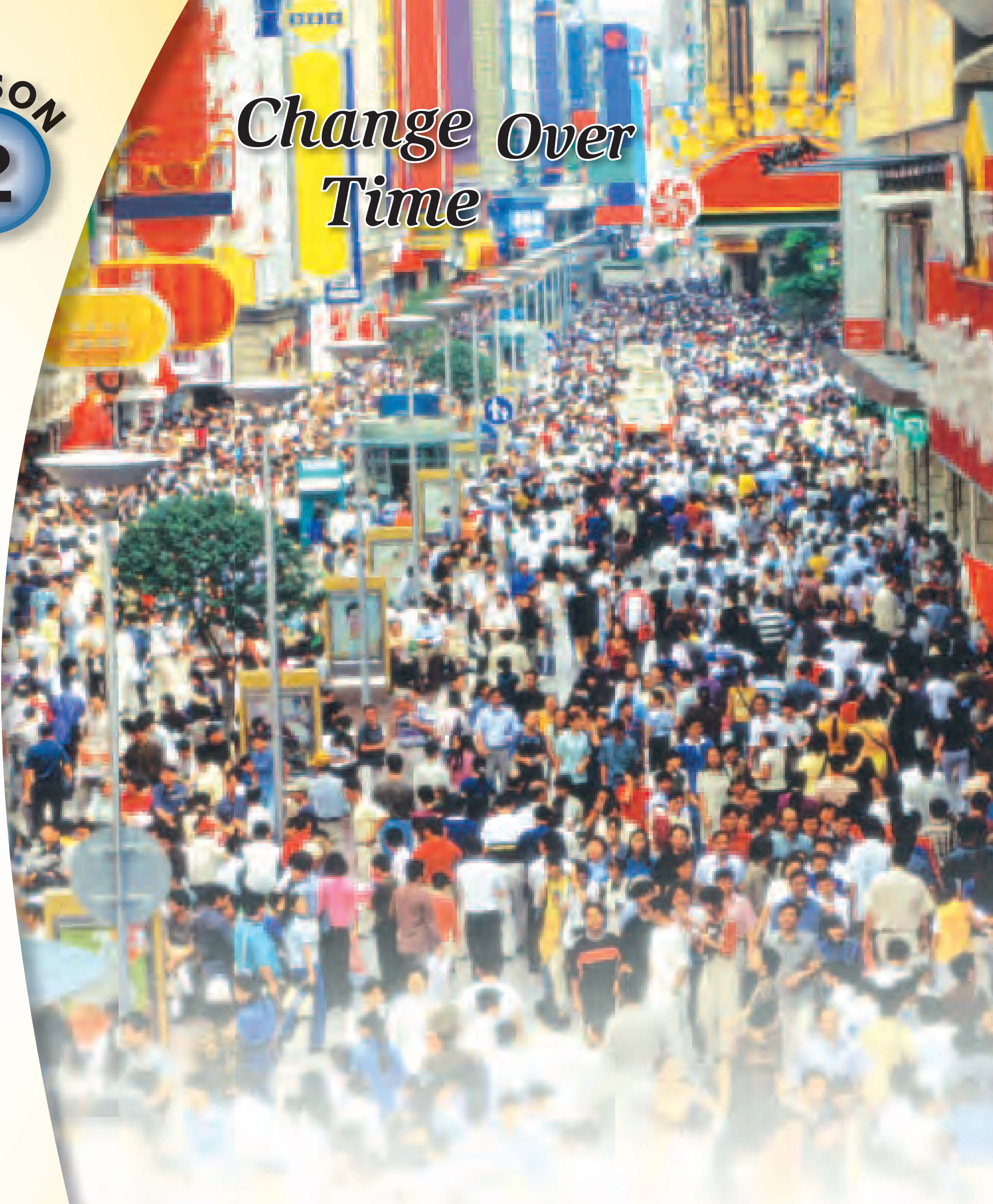
c. earns 6.5% interest over that year.

d. declines by 8.7% over the year because withdrawals exceed interest earned.

e. declines at an average rate of $2.50 per month.

LESSON 2

Change Over Time

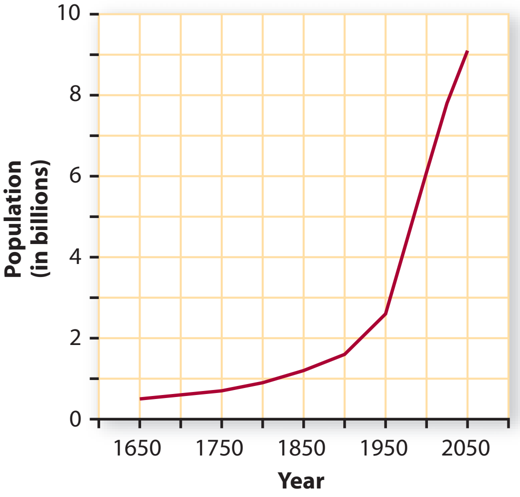


Every 10 years, the U.S. Census Bureau counts every American citizen and permanent resident. The 2000 census reported the U.S. population to be 281 million, with growth at a rate of about 1% each year. The world population is over 6 billion and growing at a rate that will cause it to exceed 9 billion by the year 2050.

Bob Krist/CORBIS

National, state, and local governments and international agencies provide many services to people across our country and around the world. To match resources to needs, it is important to have accurate population counts more often than once every 10 years. However, complete and accurate census counts are very expensive.

World Population 1650–2050



Source: www.census.gov/ipc/www/world.html

Think About This Situation

The population of the world and of individual countries, states, and cities changes over time.

a How would you describe the pattern of change in world population from 1650 to 2050?

b What do you think are some of the major factors that influence population change of a city, a region, or a country?

c How could governments estimate year-to-year population changes without making a complete census?

Your work on investigations of this lesson will develop your understanding and skill in using algebra to solve problems involving variables like populations that change as time passes.

Investigation 1 Predicting Population Change

If you study trends in population data over time, you will often find patterns that suggest ways to predict change in the future. There are several ways that algebraic rules can be used to explain and extend such patterns of change over time. As you work on the problems of this investigation, look for an answer to this question:

What data and calculations are needed to predict human and animal populations into the future?

Population Change in Brazil Brazil is the largest country in South America. Its population in the year 2005 was about 186 million.

Census statisticians in Brazil can estimate change in that country's population from one year to the next using small surveys and these facts:

- Based on recent trends, births every year equal about 1.7% of the total population of the country.
- Deaths every year equal about 0.6% of the total population.

Source: *CIA—The World Factbook 2005*

1 How much of the estimated change in Brazil's population from 2005 to 2006 was due to:

a. births? **b.** deaths? **c.** both causes combined?

2 Calculate estimates for the population of Brazil in 2006, 2007, 2008, 2009, and 2010. Record those estimates and the year-to-year changes in a table like the one below.

Population Estimates for Brazil

Year	Change (in millions)	Total Population (in millions)
2005	—	186
2006	?	?
2007	?	?
2008	?	?
2009	?	?
2010	?	?

a. Make a plot of the (*year, total population*) data.

b. Describe the pattern of change over time in population estimates for Brazil. Explain how the pattern you describe is shown in the table and in the plot.

3 Which of these strategies for estimating *change* in Brazil's population from one year to the next uses the growth rate data correctly? Be prepared to justify your answer in each case.

a. 0.017(*current population*) − 0.006(*current population*) = *change in population*

b. 0.011(*current population*) = *change in population*

c. 0.17(*current population*) − 0.06(*current population*) = *change in population*

d. 1.7%(*current population*) − 0.6% = *change in population*

4 Which of the following strategies correctly use the given growth rate data to estimate the *total* population of Brazil one year from now? Be prepared to justify your answer in each case.

a. (*current population*) + 0.011(*current population*) = *next year's population*

b. (*current population*) + 0.017(*current population*) − 0.006(*current population*) = *next year's population*

c. 1.011(*current population*) = *next year's population*

d. 186 million + 1.7 million − 0.6 million = *next year's population*

5 Use the word *NOW* to stand for the population of Brazil in any year and the word *NEXT* to stand for the population in the next year to write a rule that shows how to calculate *NEXT* from *NOW.* Your rule should begin "*NEXT* = ..." and then give directions for using *NOW* to calculate the value of *NEXT.*

The Whale Tale In 1986, the International Whaling Commission declared a ban on commercial whale hunting to protect the small remaining stocks of several whale types that had come close to extinction.

Scientists make census counts of whale populations to see if the numbers are increasing. While it's not easy to count whales accurately, research reports have suggested that one population, the bowhead whales of Alaska, was probably between 7,700 and 12,600 in 2001.

The difference between whale births and natural deaths leads to a natural increase of about 3% per year. However, Alaskan native people are allowed to hunt and kill about 50 bowhead whales each year for food, oil, and other whale products used in their daily lives.

6 Assume that the 2001 bowhead whale population in Alaska was the low estimate of 7,700.

a. What one-year change in that population would be due to the difference between births and natural deaths?

b. What one-year change in that population would be due to hunting?

c. What is the estimate of the 2002 population that results from the combination of birth, death, and hunting rates?

7 Use the word *NOW* to stand for the Alaskan bowhead whale population in any given year and write a rule that shows how to estimate the population in the *NEXT* year.

8 Which of the following changes in conditions would have the greater effect on the whale population over the next few years?

- decrease in the natural growth rate from 3% to 2%, or
- increase in the Alaskan hunting quota from 50 to 100 per year

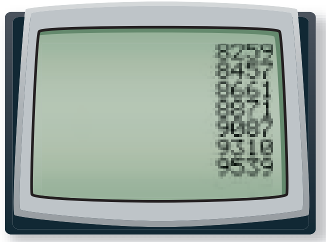
Tom Brakefield/Digital Vision/Getty Images

In studies of population increase and decrease, it is often important to predict change over many years, not simply from one year to the next. It is also interesting to see how changes in growth factors affect changes in populations. Calculators and computers can be very helpful in those kinds of studies.

For example, the following calculator procedure gives future estimates of the bowhead whale population with only a few keystrokes:

Calculator commands | **Expected display**

7700
ENTER
.03 × Answer + Answer − 50
ENTER
ENTER
ENTER
ENTER
ENTER
ENTER
ENTER
ENTER
ENTER



9 Examine the calculator procedure above.

a. What seem to be the purposes of the various keystrokes and commands?

b. How do the instructions implement a *NOW-NEXT* rule for predicting population change?

10 Modify the given calculator steps to find whale population predictions starting from the 2001 high figure of 12,600 and a natural increase of 3% per year.

a. Find the predicted population for 2015 if the annual hunt takes 50 whales each year.

b. Suppose that the hunt takes 200 whales each year instead of 50. What is the predicted population for 2015 in this case?

c. Experiment to find a hunt number that will keep the whale population stable at 12,600.

Summarize the Mathematics

In the studies of human and whale populations, you made estimates for several years based on growth trends from the past.

a What trend data and calculations were required to make these estimates:

i. The change in the population of Brazil from one year to the next? The new total population of that country?

ii. The change in number of Alaskan bowhead whales from one year to the next? The new total whale population?

b What does a *NOW-NEXT* rule like $NEXT = 1.03 \cdot NOW - 100$ tell about patterns of change in a variable over time?

c What calculator commands can be used to make population predictions for many years in the future? How do those commands implement *NOW-NEXT* rules?

Be prepared to share your thinking with the class.

Check Your Understanding

The 2000 United States Census reported a national population of about 281 million, with a birth rate of 1.4%, a death rate of 0.9%, and net migration of about 1.1 million people per year. The net migration of 1.1 million people is a result of about 1.3 million immigrants entering and about 0.2 million emigrants leaving each year.

a. Use the given data to estimate the U.S. population for years 2001, 2005, 2010, 2015, 2020.

b. Use the words *NOW* and *NEXT* to write a rule that shows how to use the U.S. population in one year to estimate the population in the next year.

c. Write calculator commands that automate calculations required by your rule in Part b to get the U.S. population estimates.

d. Modify the rule in Part b and the calculator procedure in Part c to estimate U.S. population for 2015 in case:

i. The net migration rate increased to 1.5 million per year.

ii. The net migration rate changed to −1.0 million people per year. That is, if the number of emigrants (people leaving the country) exceeded the number of immigrants (people entering the country) by 1 million per year.

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Investigation 2 Tracking Change with Spreadsheets

One of the most useful tools for exploring relations among birth rates, death rates, migration rates, and population totals is a computer *spreadsheet.*

A spreadsheet is an electronic grid of cells in which numerical data or labels can be stored. The cells of a spreadsheet can be related by formulas, so that the numerical entry of one cell can be calculated from data in other cells.

The following table shows a piece of one spreadsheet that predicts growth of the Alaskan bowhead whale population.

Whale Population.xls

	A	B	C
1	Year	Population	Natural Growth Rate
2	2001	7700	1.03
3	2002	7881	Hunting Rate
4	2003	8067	50
5	2004	8259	
6	2005	8457	
7			

As you work on the problems in this investigation, think about the following question:

How do basic spreadsheet methods use the NOW-NEXT way of thinking to help solve problems about change over time?

1 From your earlier work with calculators, the numbers in column B of the preceding spreadsheet probably look familiar. However, you can't see how the spreadsheet actually produced those numbers. The next table shows the formulas used to calculate entries in columns A and B of the first display.

Whale Population.xls

A6 =A5+1

	A	B	C
1	Year	Population	Natural Growth Rate
2	2001	7700	1.03
3	=A2+1	=1.03*B2−50	Hunting Rate
4	=A3+1	=1.03*B3−50	50
5	=A4+1	=1.03*B4−50	
6	=A5+1	=1.03*B5−50	
7			

Compare the formula cell entries to the numerical cell values in the display above to help answer the next questions about how spreadsheets actually work.

a. How do you think the formulas in cells A3, A4, A5, and A6 produce the pattern of entries 2002, 2003, 2004, and 2005 in the numerical form of the spreadsheet?

b. How do you think the formulas in cells B3, B4, B5, and B6 produce the pattern of entries 7881, 8067, 8259, and 8457?

c. Why would it make sense to call the formulas in cells A3–A6 and B3–B6 *NOW-NEXT* formulas?

d. What are the starting values for the formulas in columns A and B?

The real power of a spreadsheet comes from a feature not shown in this table of formulas. After entering the starting values in cells A2 and B2 and the *NOW-NEXT* formulas in cells A3 and B3, the spreadsheet command "fill down" will automatically produce formulas for the cells below, changing the cell reference A2 to A3, B2 to B3, and so on.

2 Suppose that you were interested in studying population growth of the United States in 10-year intervals corresponding to the national census counts. With the 2000 population of 281 million, a natural 10-year growth rate of about 5%, and 10-year migration of about 11 million, a spreadsheet to make predictions for several decades might begin like the one below.

U.S. Population.xls

B3 =1.05*B2+11

	A	B	C
1	Year	Population	Natural Growth Rate
2	2000	281	1.05
3	=A2+10	=1.05*B2+11	Migration Rate
4			11
5			
6			

a. What formula and numerical entries would you expect in cells A3, A4, A5, and A6 if you use a fill down command in that column?

b. What formula and numerical entries would you expect in cells B3, B4, B5, and B6 if you use a fill down command in that column?

A second feature of spreadsheets makes exploratory work even more efficient. If you mark column and/or row labels with a dollar sign symbol, they will not change in response to fill down or fill across commands.

3 Suppose that you want to study the effects of change in both natural growth and migration rates for the U.S. population.

a. What numerical value do you think will result from the formula "=C$2*B2+C$4" in cell B3 of the spreadsheet below?

U.S. Population.xls

B3 =C$2*B2+C$4

	A	B	C
1	Year	Population	Natural Growth Rate
2	2000	281	1.05
3	=A2+10	=C$2*B2+C$4	Migration Rate
4	=A3+10		11
5	=A4+10		
6			

b. What formulas and numerical values will appear in cells B4 and B5 following a fill down command?

c. What formulas and numerical values will appear in cells B4 and B5 if the entry in cell C2 is changed to 1.06 and the entry in cell C4 is changed to 12?

d. What changes in natural growth and migration rates are implied by those changes in the spreadsheet?

4 When Robin got a summer job, she decided she could save $25 from her pay every week.

a. Construct a spreadsheet that will display Robin's total savings at the end of each week during the 10-week summer job.

b. If necessary, modify your spreadsheet so that the amount saved each week can be found by changing only one cell entry. Then use the new spreadsheet to display Robin's total savings at the end of each week if she actually saves only $17.50 per week.

5 Suppose that in September, Robin invests her summer savings of $250 in a bank account that pays interest at the rate of 0.5% per month (an annual rate of 6%).

a. Construct a spreadsheet that will display Robin's bank balance at the end of each month for the next year.

b. Modify your spreadsheet to account for Robin's habit of withdrawing $20 at the beginning of every month for extra spending money.

6 Modify the spreadsheet in Problem 5 to compare two possible savings plans.

Plan 1: Deposit $100 in September and add $10 per month thereafter.
Plan 2: Deposit $0 in September and add $20 per month thereafter.

a. How long will it take before Plan 2 gives a greater balance than Plan 1?

b. How will the answer to Part a change if the monthly interest rate decreases to 0.4%, 0.3%, 0.2%, or 0.1%?

7 When José was considering purchase of a $199 portable music player and ear phones, he was told that resale value of the gear would decline by about 5% per month after he bought it.

a. Construct a spreadsheet that will display the value of José's music gear at the start of each month over two years from its purchase.

b. Modify your spreadsheet to analyze the changing value of a PDA that would cost $499 to purchase and decline in value at about the same percent rate.

c. Explain why your spreadsheets in Parts a and b do not show loss of all value for the music player or the PDA in 20 months, even though $20 \cdot 5\% = 100\%$.

Tim Pannell/CORBIS

Summarize the Mathematics

In this investigation, you learned basic spreadsheet techniques for studying patterns of change.

a. How are cells in a spreadsheet grid labeled and referenced by formulas?

b. How are formulas used in spreadsheets to produce numbers from data in other cells?

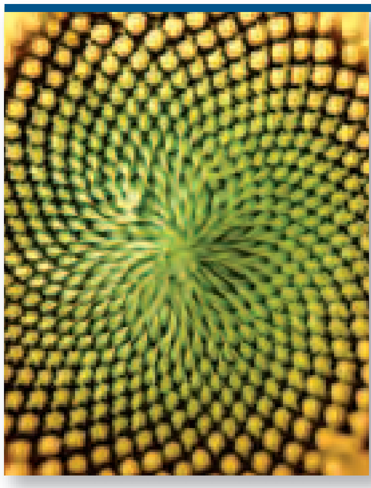
c. How is the “fill” command used to produce cell formulas rapidly?

d. How are the cell formulas in a spreadsheet similar to the *NOW-NEXT* rules you used to predict population change?

Be prepared to share your ideas with other students.

Check Your Understanding

The number pattern that begins 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, … is known as the **Fibonacci sequence**. The pattern appears many places in nature. It also has been the subject of many mathematical investigations.



a. Study the pattern. What are the next five numbers in the sequence?

b. Write spreadsheet formulas that will produce columns A and B in the next table (and could be extended down to continue the pattern).

Fibonacci Sequence.xls

◇	A	B
1	1	1
2	2	1
3	3	2
4	4	3
5	5	5
6	6	8
⋮	⋮	⋮

c. Modify the spreadsheet of Part b to produce terms in the number pattern that begins 5, 5, 10, 15, 25, … and grows in the Fibonacci way. Use the spreadsheet to find the next 10 numbers in the pattern.

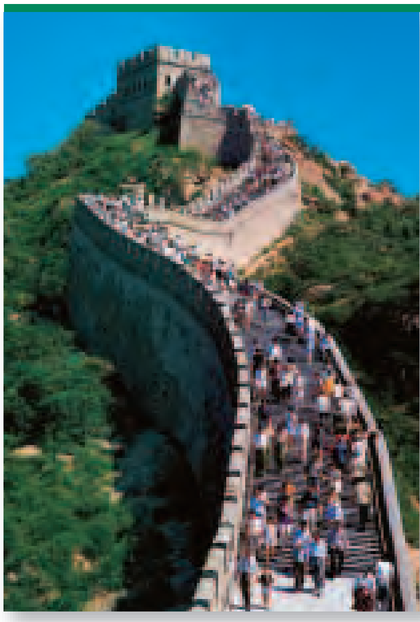
d. Compare the number patterns in Parts a and c. What explains the way the patterns are related?

(t)Robert Essel NYC/CORBIS, (b)Steve Terrill

On Your Own

Applications

1 The People's Republic of China is the country with the largest population in the world. The population of China in 2005 was approximately 1.3 billion. Although families are encouraged to have only one child, the population is still growing at a rate of about 0.6% per year.



a. Estimate the population of China for each of the next 5 years and record your estimates in a data table.

b. When is it likely that the population of China will reach 1.5 billion?

c. How would your prediction in Part b change if the growth rate were 1.2%, double the current rate?

d. Using the word *NOW* to stand for the population in any year, write rules that show how to calculate the population in the *NEXT* year:

i. if the growth rate stays at 0.6%.

ii. if the growth rate doubles to 1.2%.

2 The country with the second largest population in the world is India, with about 1.1 billion people in 2005. The birth rate in India is about 2.2% per year and the death rate is about 0.8% per year.

a. Estimate the population of India for each of the next 5 years and record your estimates in a data table.

b. When is it likely that the population of India will reach 1.5 billion?

c. How would your prediction in Part b change if the birth rate slows to 2.0%?

d. Using the word *NOW* to stand for the population in any year, write rules that show how to calculate the population in the *NEXT* year:

i. if the birth rate stays at 2.2%.

ii. if the birth rate slows to 2.0%.

Panorama Stock/Alamy Images

3 Timber wolves were once very common in wild land across the northern United States. However, when the Endangered Species Act was passed in 1973, wolves were placed on the endangered list.

Thirty years later, the wolf populations have recovered in the northern Rockies and in the forests of Minnesota, Wisconsin, and Michigan.

In 2003, estimates placed the Midwest wolf population at more than 3,100 with an annual growth rate of 25% to 30%. (Source: "Timber Wolves Resurgent in Upper Midwest," *The Washington Post*, Monday, February 10, 2003.)

a. Use the given wolf population estimate and the 25% growth rate to predict populations for 10 years (from 2003 to 2013). Record your results in a data table.

b. Estimate the time when the Midwest wolf population will reach 30,000 (the number believed to have lived in the Great Lakes region 500 years ago).

c. How does your answer to Part b change if you use the higher growth rate estimate of 30%?

d. Using the word *NOW* to stand for the Midwest wolf population in any year, write rules that show how to calculate the population in the *NEXT* year:

- **i.** if the growth rate stays at 25%.
- **ii.** if the growth rate increases to 30%.

4 Midwestern farmers who raise dairy cattle are concerned that growing wolf populations described in Task 3 above threaten the safety of their herds. They want permission to eliminate wolves that kill livestock.

a. Make a table showing how the Midwest wolf population of 3,100 would change over the next 10 years if an annual harvest of 250 animals were allowed, but the natural growth rate continued at 25% per year.

b. When is it likely that the Midwest wolf population would reach 30,000 if the annual harvest of 250 animals were permitted?

c. How would your answer to Part b change if the annual harvest were increased to 500?

d. Using the word *NOW* to stand for the population in any year, write rules that show how to calculate the population in the *NEXT* year:

- **i.** if the natural growth rate stays at 25% and 250 wolves are killed each year.
- **ii.** if the growth rate stays at 25% but the annual harvest increases to 500 wolves per year.

Alan and Sandy Carey/CORBIS

5. China experiences annual negative net migration of its population. People leave for other countries of the world in large numbers.

   a. How would the current 1.3 billion population of China change in 10 years in case of natural growth rate of 0.6% and net migration of about −500,000 people per year? (Remember to use uniform units.)

   b. What net migration would have to occur for China to reach *zero population growth,* assuming that the natural growth rate remained at 0.6% per year?

   c. Using *NOW* to stand for the population of China in any year, write a rule that shows how to calculate the population in the *NEXT* year if the natural growth rate is 0.6% and the net migration is about −500,000 people per year.

6. India has an annual negative migration to somewhat offset its natural population growth.

   a. How would the current 1.1 billion population of India change in 10 years in case of a natural growth rate of 1.4% and net migration of about only −80,000 people per year? (Use uniform units.)

   b. What net migration would have to occur for India to reach *zero population growth,* assuming that the natural growth rate remained at 1.4% per year?

   c. Using *NOW* to stand for the population of India in any year, write a rule that shows how to calculate the population in the *NEXT* year if the natural growth rate is 1.4%, and the net migration is about −80,000 people per year.

7. If money is invested in a savings account, a business, or real estate, its value usually increases each year by some percent. For example, investment in common stocks yields growth in value of about 10% per year in the long run. Suppose that when a child is born, the parents invest $1,000 in a mutual fund account.

   a. If that fund actually grows in value at a rate of 10% per year, what will its value be after 1 year? After 2 years? After 5 years? After 18 years when the child is ready to go to college?

   b. Using *NOW* to stand for the investment value at the end of any year, write a rule showing how to calculate the value at the end of the *NEXT* year.

   c. How will your answers to Parts a and b change if:

      i. the initial investment is only $500?

      ii. the initial investment is $1,000, but the growth rate is only 5% per year?

   d. How will your answers to Parts a and b change if, in addition to the percent growth of the investment, the parents add $500 per year to the account?

8. Select one of Applications Tasks 1–3 and develop a spreadsheet that could be used to answer the population growth questions asked in those items. Use the spreadsheet to answer those questions.

9 Select one of Applications Tasks 4–7 and develop a spreadsheet that could be used to answer the questions about population or investment growth over time. Use the spreadsheet to answer those questions.

Connections

Data in the next table show population (in thousands) of some major U.S. cities in 1990 and in 2000. Use the data to complete Connections Tasks 10–13 that follow.

Major U.S. Cities: 1990 and 2000 Population (in 1,000s)

U.S. City	1990	2000	U.S. City	1990	2000
Atlanta, GA	394	416	Independence, MO	112	113
Aurora, CO	222	276	Milwaukee, WI	628	597
Berkeley, CA	103	103	Newark, NJ	275	274
Boise, ID	127	186	Portland, OR	437	529
El Paso, TX	515	564	St. Louis, MO	397	348
Hartford, CT	140	122	Washington, DC	607	572

10 The population of Berkeley, California, changed by fewer than 1,000 people. Among the remaining cities in the list, which cities had:

a. the greatest absolute decrease in population?

b. the greatest absolute increase in population?

c. the greatest percent decrease in population?

d. the greatest percent increase in population?

11 What were the mean and median population change for the listed cities?

12 Suppose that the population of Aurora, Colorado, continues increasing at the rate it changed between 1990 and 2000.

a. What population for Aurora would be predicted for 2010, 2020, 2030, 2040, and 2050 if population increases by the same number of people in each decade?

b. What *NOW-NEXT* rule describes the pattern of change in Part a?

c. What was the percent change in the Aurora population between 1990 and 2000?

d. What *NOW-NEXT* rule describes the pattern of change in Aurora's population each decade, if the percent rate of change from Part c is used?

e. What population is predicted for Aurora in 2010, 2020, 2030, 2040, and 2050 if growth occurs at the percent rate of Part c?

f. How are the predicted change patterns in Parts a and e similar, and how are they different? Why are they not the same?

g. What reasons could you have to doubt the predictions of Parts a or e?

13 Suppose that the population of Washington, D.C., continues decreasing at the rate it changed between 1990 and 2000.

a. What population for Washington, D.C., would be predicted for 2010, 2020, 2030, 2040, and 2050 if population decreases by the same number of people in each decade?

b. What *NOW-NEXT* rule describes the pattern of change in Part a?

c. What was the percent change in the Washington, D.C., population between 1990 and 2000?

d. What *NOW-NEXT* rule describes the pattern of change in the Washington, D.C., population each decade, if the percent rate of change from Part c is used?

e. What population is predicted for Washington, D.C., in 2010, 2020, 2030, 2040, and 2050 if growth occurs at the percent rate of Part c?

f. How are the predicted change patterns in Parts a and e similar, and how are they different? Why are they not the same?

g. What reasons could you have to doubt the predictions of Parts a or e?

14 Sketch graphs that match each of the following stories about quantities changing over time. On each graph, label the axes to indicate reasonable scale units for the independent variable and the dependent variable. For example, use "time in hours" and "temperature in degrees Fahrenheit" for Part a.

a. On a typical summer day where you live, how does the temperature change from midnight to midnight?

b. When a popular movie first appears in video rental stores, demand for rentals changes as time passes.

c. The temperature of a cold drink in a glass placed on a kitchen counter changes as time passes.

d. The number of people in the school gymnasium changes before, during, and after a basketball game.

Jeff Greenberg/PhotoEdit

15 Each part below gives a pair of *NOW-NEXT* rules. For each rule in each pair, produce a table of values showing how the quantities change from the start through 5 stages of change. Then compare the patterns of change produced by each rule in the pair and explain how differences are related to differences in the *NOW-NEXT* rules.

a. Rule 1: $NEXT = NOW + 10$, starting at 5
Rule 2: $NEXT = NOW + 8$, starting at 5

Sample Table:

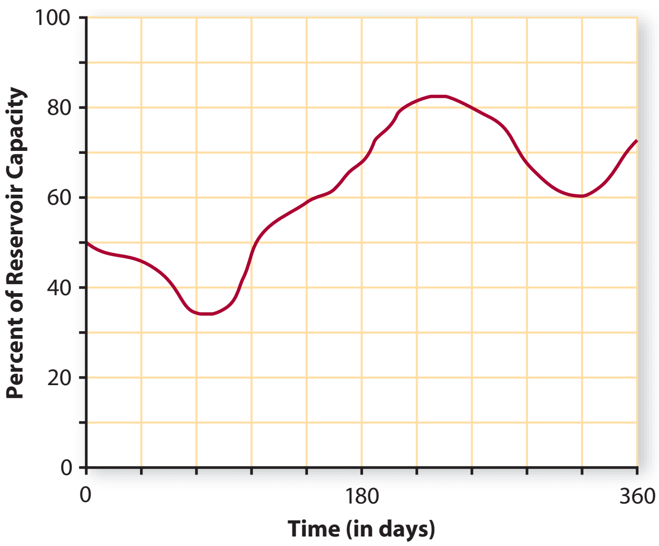
Stage	0	1	2	3	4	5
Rule 1	5	15	25			
Rule 2	5	13	...			

b. Rule 1: $NEXT = 2 \cdot NOW$, starting at 5
Rule 2: $NEXT = 1.5 \cdot NOW$, starting at 10

c. Rule 1: $NEXT = 0.5 \cdot NOW$, starting at 100
Rule 2: $NEXT = 0.9 \cdot NOW$, starting at 50

d. Rule 1: $NEXT = 2 \cdot NOW + 10$, starting at 8
Rule 2: $NEXT = 3 \cdot NOW - 10$, starting at 8

16 The graph below shows how the amount of water in a city's reservoir changed during one recent year.



On a copy of the graph, mark points that show when the reservoir's water supply is:

a. increasing at the fastest rate—label the point(s) with the letter "A".

b. decreasing at the fastest rate—label the point(s) with the letter "B".

c. increasing at a constant rate—label the point(s) with the letter "C".

d. decreasing at a constant rate—label the point(s) with the letter "D".

e. neither increasing nor decreasing—label the point(s) with the letter "E".

Reflections

17 How are patterns in the data tables and graphs arising in the studies of human and whale populations similar to or different from those that related:

a. *weight* and *stretched length* of a bungee cord (page 5)?

b. *price per jump* and *number of customers* for a bungee jump (page 6)?

c. *price per jump* and *daily income* for operation of the bungee jump (page 6)?

d. *number of plays* and fund-raiser *cumulative profit* in the *Take a Chance* die-tossing game (pages 8–9)?

e. *average speed* and *race time* for a 500-mile NASCAR race (pages 11–12)?

f. *hours worked* and *earnings* at Fresh Fare Market (page 12)?

18 In what ways are the methods used to describe "change over time" patterns similar to or different from the methods used to study "cause and effect" patterns?

19 Consider the *NOW-NEXT* rules:

$$NEXT = NOW + 0.05 \cdot NOW \quad \text{and} \quad NEXT = 1.05 \cdot NOW$$

a. Find several values produced by these *NOW-NEXT* rules, starting from $NOW = 10$.

b. Then explore the patterns produced by each rule for some other common starting values.

c. Explain why the results of the explorations in Parts a and b are not surprising.

20 Do the ideas of independent and dependent variables have useful meaning in the study of "change over time" patterns? If so, how? If not, why not?

21 Both animal and human population growth rates commonly change as the years pass.

a. What factors might cause change in the percent growth rates of a population?

b. Why, if growth rates change, does it still make sense to use current growth rates for predictions of future populations?

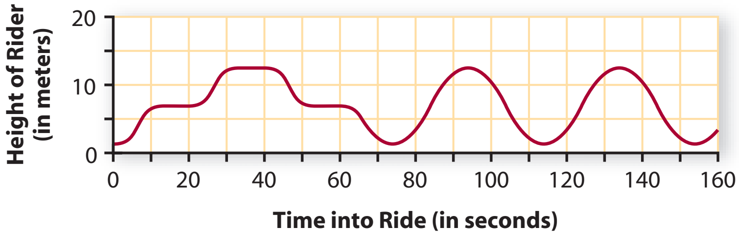
Extensions

22 The amusement park ride test team took their radar gun for a ride on the Ferris wheel. They aimed the gun at the ground during two nonstop trips around on the wheel, giving a graph relating height above the ground to time into the trip.

a. The total time for the ride was 100 seconds. Sketch a graph showing how you think height will change over time during the ride. Then write an explanation of the pattern in your graph. (*Hint:* You might experiment with a bicycle wheel as a model of a Ferris wheel; as you turn the wheel, how does the height of the air valve stem change?)

b. Given next is the graph of (*time into ride, height of rider*) data for one Ferris wheel test ride. Write an explanation of what the graph tells about that test ride.

First Test Ride



c. Given next are some (*time, height*) data from a second test ride. Write a short description of the pattern of change in height over time during this ride.

Second Test Ride

Time (in seconds)	0	2	5	10	15	20	22.5	25	30	35	40	42.5	45	50	55	60	62.5	65
Height (in meters)	1	1	3	3	11	11	13	11	11	3	3	1	3	3	11	11	13	11

d. Plot the data from the second test ride on a coordinate graph. Connect the points if it seems to make sense to do so. Then explain whether you think the graph or the table better shows the pattern of change in height during the ride.

Travel Ink Photo/Index Stock

23 A manatee is a large sea mammal native to Florida waters that is listed as endangered. The chart below gives the number of manatees killed in watercraft collisions near the Gulf Coast of Florida every year from 1985 through 2004.

Manatee/Watercraft Mortalities

Year	Number of Manatees Killed	Year	Number of Manatees Killed
1985	33	1995	42
1986	33	1996	60
1987	39	1997	54
1988	43	1998	66
1989	50	1999	82
1990	47	2000	78
1991	53	2001	81
1992	38	2002	95
1993	35	2003	73
1994	49	2004	69

Source: www.savethemanatee.org/mortalitychart.htm

a. Prepare a plot of the number of manatees killed in watercraft collisions between 1985 and 2004. Connect the points in that plot to help you study trends in manatee/watercraft mortalities.

b. Describe the pattern of change in mortalities that you see in the table and the *plot over time*.

c. During what one-year period was there the greatest change in manatee deaths if one measures change by:

  i. difference?

  ii. percent change?

d. How is the time of greatest change in manatee deaths shown in the table? In the graph?

e. What factors in the marine life and boating activity of Florida might be causing the increase in manatee deaths? What actions could be taken by government to reduce the number of deaths?

Douglas Faulkner/Photo Researchers

24 The Fibonacci sequence has many interesting and important properties. One of the most significant is revealed by studying the ratios of successive terms in the sequence. Consider the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, … .

a. Modify the spreadsheet you wrote to generate terms of the Fibonacci sequence to include a new column C. In cell C2, enter the formula "=B2/B1" and then repeat this formula (with cell references changing automatically down the column C). Record the sequence of terms generated in that column.

b. What pattern of numerical values do you notice as you look farther and farther down column C?

Leonardo Fibonacci
1180–1250

25 In an earlier problem, you explored the rate at which the allowance paid to Ethan and Anna would increase if it began with only 1 penny on the first day of the month but doubled each day thereafter.

a. Write a spreadsheet with three columns: "Day of the Month" in column A, "Daily Allowance" in column B, and "Cumulative Allowance" in column C, with rows for up to 31 days.

b. Use the spreadsheet to find the total allowance paid to Ethan and Anna in a month of 31 days.

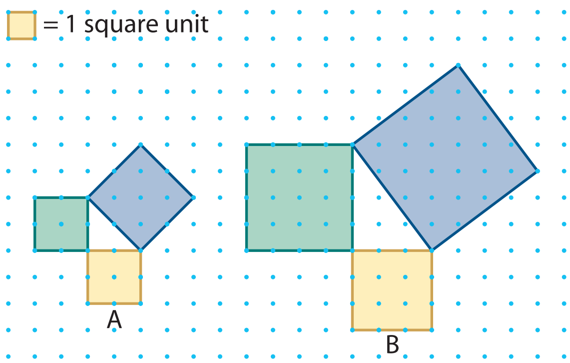
Review

26 Find the value of each expression.

a. $4 \cdot 2 - 3$ **b.** $2(-5) + 2(3)$ **c.** $-5^2 - (3 - 5)$

d. $(-3)(2)(-5)$ **e.** $-5 + 2 + 10 + (-5)^2$ **f.** $|-5| + 15 - |5 - 3|$

27 In figures A and B, squares are built on each side of a right triangle.



a. For figure A:

i. Find the area of the triangle.

ii. Find the area of each square. How are the areas related?

iii. Find the perimeter of the triangle.

b. For figure B:

i. Find the area of the triangle.

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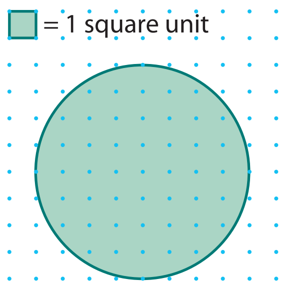
ii. Find the area of each of the squares. How are the areas of the squares related in this case?

iii. Find the perimeter of the triangle.

c. How is the work you did in Parts a and b related to the Pythagorean Theorem?

28 Consider the circle drawn below.



a. Use the dot grid to find the approximate area of the circle.

b. Use the formula $C = 2\pi r$ or $C = \pi d$ to find the circumference of the circle.

c. Use the formula $A = \pi r^2$ to find the area of the circle.

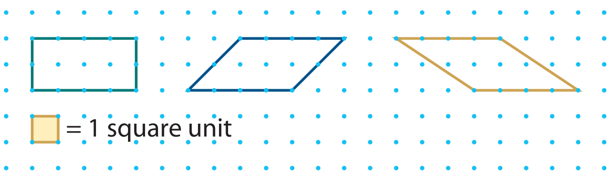
d. In what kind of unit is area measured? How can you use this fact to avoid confusing the formulas $2\pi r$ and πr^2 when computing the area of a circle?

29 The population of India is about 1.1 billion people. Suppose the population of country X is 1.1 million, and the population of country Y is 11 million.

a. How many times larger is the population of India than that of country Y? Than that of country X?

b. What percent of the Indian population is the population of country Y? Is the population of country X?

30 Consider the three parallelograms shown below.



a. Rachel thinks that all three parallelograms have the same area. Is she correct? Explain your reasoning.

b. Sketch a parallelogram, different from those above, that has an area of 8 square units.

31 Place the following numbers in order from smallest to largest.

2.25 2.05 −2.35 −2.75 0 2.075

LESSON 3

Tools for Studying Patterns of Change

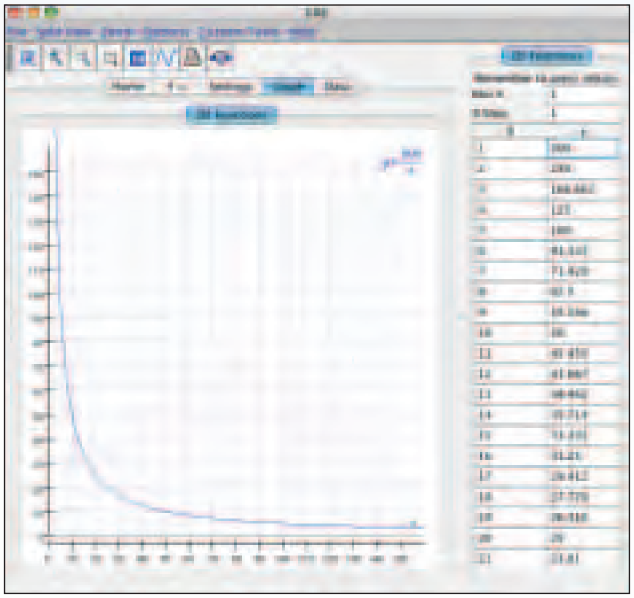
In your work on problems of Lesson 1, you studied a variety of relationships between dependent and independent variables. In many cases, those relationships can be expressed by calculating rules that use letter names for the variables. For example,

The *stretched length* L of a simulated bungee cord depends on the *attached weight* w in a way that is expressed by the rule $L = 30 + 0.5w$.

The *number of customers* n for a bungee jump depends on the *price per jump* p in a way that is expressed by the rule $n = 50 - p$.

The *time* t of a 500-mile NASCAR race depends on the *average speed* s of the winning car in a way that is expressed by the rule $t = \frac{500}{s}$.

These symbolic rules give directions for calculating values of dependent variables from given values of related independent variables. They also enable use of calculator and computer tools for solving problems involving the relationships.



Think About This Situation

If you were asked to solve problems in situations similar to those described on the previous page:

a How would you go about finding algebraic rules to model the relationships between dependent and independent variables in any particular case?

b What ideas do you have about how the forms of algebraic rules are connected to patterns in the tables and graphs of the relationships that they produce?

c How could you use calculator or computer tools to answer questions about the variables and relationships expressed in rules?

Your work in this lesson will help you develop skills in writing algebraic rules to express relationships between variables. You will also use calculator and computer strategies to determine relationships expressed by those rules.

Investigation 1 Communicating with Symbols

The first challenge in using algebraic expressions and rules to study a relationship between variables is to write the relationship in symbolic form. There are several ways information about such relationships occur and several strategies for translating information into symbolic form. As you work on the problems of this investigation, look for answers to this question:

What are some effective strategies for finding symbolic expressions that represent relationships between variables?

Translating Words to Symbols In many problems, important information about a relationship between variables comes in the form of written words. Sometimes it is easy to translate those words directly into algebraic expressions.

For example, if a restaurant adds a 15% gratuity to every food bill, the *total bill* T is related to the *food charges* F by the rule:

$$T = F + 0.15F.$$

In other cases, you might need to calculate the value of the dependent variable for several specific values of the independent variable to see how the two are related in general. Suppose that a library loans books free for a week, but charges a fine of \$0.50 each day the book is kept beyond the first week. To find a rule relating *library fines* for books to the *number of days* the book is kept, you might begin by calculating some specific fines, like these:

Book Kept 10 Days: Fine $= 0.50(10 - 7)$
Book Kept 21 Days: Fine $= 0.50(21 - 7)$

1 Can you create a rule relating *library fines* for new books to the *number of days* the book is kept? Write your rule in symbolic form, using F for the fine and d for number of days the book is kept.

2 Midwest Amusement Park charges $25 for each daily admission. The park has daily operating expenses of $35,000.

a. What is the operating profit (or loss) of the park on a day when 1,000 admission tickets are sold? On a day when 2,000 admission tickets are sold?

b. Write a symbolic rule showing how daily profit P for the park depends on the number of paying visitors n.

3 A large jet airplane carries 150,000 pounds of fuel at takeoff. It burns approximately 17,000 pounds of fuel per hour of flight.

a. What is the approximate amount of fuel left in the airplane after 3 hours of flight? After 7 hours of flight?

b. Write a rule showing how the amount of fuel F remaining in the plane's tanks depends on the elapsed time t in the flight.

4 The costs for a large family reunion party include $250 for renting the shelter at a local park and $15 per person for food and drink.

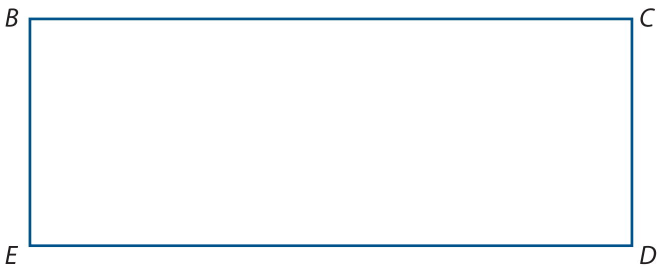
a. Write a rule showing how the total cost C for the reunion party depends on the number of people n who will attend.

b. Write another rule showing how the cost per person c (including food, drink, and a share of the shelter rent) depends on the number of people n who plan to attend.

Measurement Formulas

Many of the most useful symbolic rules are those that give directions for calculating measurements of geometric figures. You probably know several such formulas from prior mathematics studies.

5 Figure *BCDE* below is a rectangle.



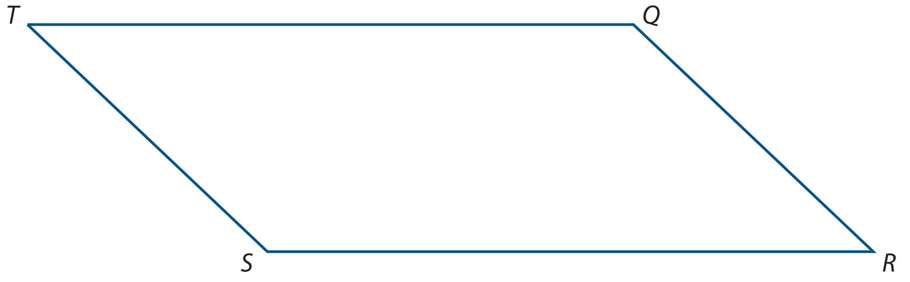
a. Use a ruler to make measurements from which you can estimate the perimeter and area of rectangle *BCDE*. Then calculate those estimates.

Kim Karpeles

b. For any given rectangle, what is the minimum number of ruler measurements you would need in order to find both its perimeter and area? What set(s) of measurements will meet that condition?

c. What formulas show how to calculate perimeter P and area A of a rectangle from the measurements described in Part b?

6 Figure *QRST* below is a parallelogram.

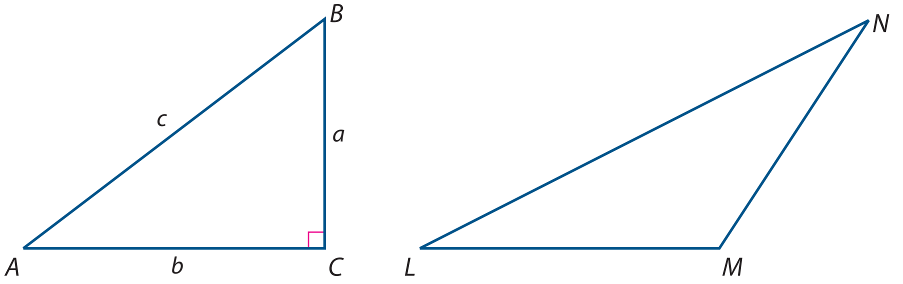


a. Use a ruler to make measurements from which you can estimate the perimeter and the area of *QRST.* Then calculate those estimates.

b. For any given parallelogram, what is the minimum number of measurements you would need in order to find both the perimeter and area? What measurements will meet that condition?

c. What formulas show how to calculate perimeter P and area A of a parallelogram from the measurements you described in Part b?

7 The two figures below are triangles—one a right triangle and one an obtuse triangle.



a. Use a ruler to make measurements from which you can estimate the perimeter and the area of each triangle. Then calculate those estimates.

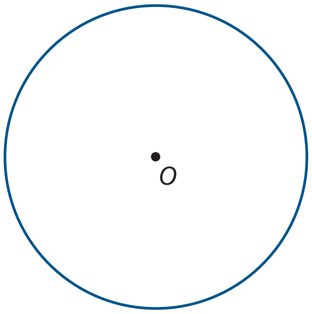
b. If the lengths of the sides of a right triangle are a, b, and c, with the side of length c opposite the right angle, the **Pythagorean Theorem** guarantees that $a^2 + b^2 = c^2$. Using this fact, what is the minimum number of ruler measurements you would need in order to find both the perimeter and area of any right triangle? What set(s) of measurements will meet that condition?

c. What formulas show how to calculate perimeter P and area A of a right triangle from the measurements you described in Part b?

d. What is the minimum number of measurements you would need in order to find both the perimeter and the area of any nonright triangle? What measurements will meet that condition?

e. What formulas show how to calculate perimeter P and area A of a nonright triangle from the measurements you described in Part d?

8 The figure below is a circle with center O.



a. Use a ruler to make measurements from which you can estimate the circumference and area of the circle. Then calculate those estimates.

b. For any given circle, what is the minimum number of measurements you would need in order to find both the circumference and the area? What measurements will meet that condition?

c. What formulas show how to calculate circumference C and area A of a circle from the measurements you described in Part b?

Summarize the Mathematics

In this investigation, you developed your skill in finding symbolic rules for patterns that relate dependent and independent variables.

a What strategies for finding algebraic rules do you find helpful when information about the pattern comes in the form of words describing the relationship of the variables?

b In general, what information is needed to calculate perimeter and area for:

i. a rectangle?
ii. a parallelogram that is not a rectangle?
iii. a right triangle?
iv. a nonright triangle?
v. a circle?

c What formulas guide calculations of perimeter and area for each figure listed in Part b?

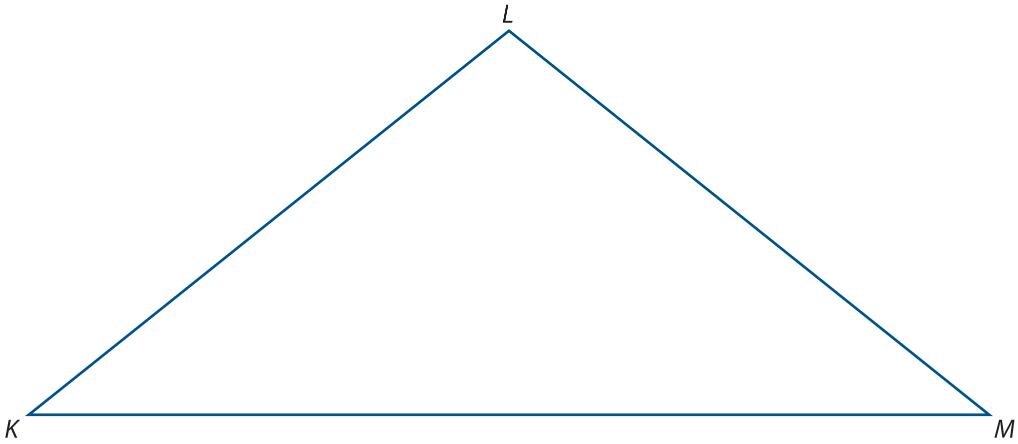
Be prepared to share your strategies and results with the class.

Check Your Understanding

Write algebraic rules expressing the relationships in these situations.

a. Students and parents who attend the Banneker High School *Take a Chance* carnival night spend an average of $12 per person playing the various games. Operation of the event costs the student government $200. What is the relationship between profit P of the carnival night and number of people who attend n?

b. The figure drawn below is an isosceles triangle with $KL = LM$.



i. Use a ruler to make measurements from which you can estimate the perimeter and area of triangle KLM.

ii. For any isosceles triangle, what is the minimum number of measurements you would need in order to estimate both its perimeter and its area? What measurements will meet that condition?

iii. Write formulas showing how the measurements you described in part ii can be used to calculate perimeter P and area A of an isosceles triangle.

Investigation 2 Quick Tables, Graphs, and Solutions

The rule $I = p(50 - p)$ predicts daily bungee jump income at Five Star Amusement Park. This rule arises from the fact that income is computed by multiplying price by the number of customers. In this case, p is the price per jump and $(50 - p)$ is the number of customers expected at that price. In this investigation, you will use rules to produce tables, graphs, and symbolic manipulations that help to answer questions such as:

- What income is expected if the price is set at $10 per jump?
- What should the price be in order to get income of at least $500 per day?
- What price per jump will produce maximum daily income?

Sometimes you can answer questions like these by doing some simple arithmetic calculations. In other cases, calculators and computers provide useful tools for the work. As you work through the investigation, look for answers to this question:

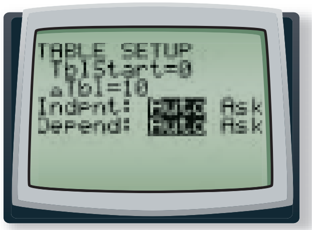
How can you use calculator or computer tools to produce tables, graphs, and symbolic manipulations, which can help you to study relationships between variables?

1 **Using Tables** You can use computer software or a graphing calculator to produce tables of related values for the independent and dependent variables. For example, examine the table of sample (*price per jump, daily income*) data below.

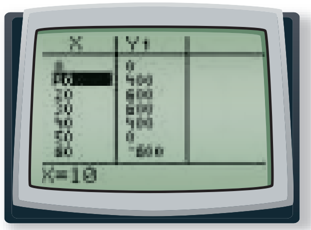
Producing a Table

Enter Rule

Set Up Table



Display Table



Scanning the table you can see, for example, that with the price set at \$10, Five Star expects a daily bungee jump income of \$400.

Use the software or calculator you have to produce and scan tables for the rule $I = p(50 - p)$ in order to estimate answers for these questions:

a. What daily income will result if the price is set at \$19?

b. To reach a daily income of at least \$500, why should the price be at least \$14, but not more than \$36?

c. What price(s) will yield a daily income of at least \$300?

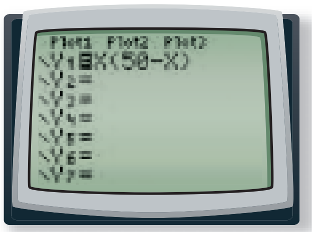
d. What price will yield the maximum possible daily income?

e. How would you describe the pattern of change in income as price increases from \$0 to \$50 in steps of \$1?

2 **Using Graphs** Computer software and graphing calculators can also be used to produce graphs of relationships between variables. For example, see the graph below for $I = p(50 - p)$ relating *price per jump* and *daily income* for the Five Star bungee jump.

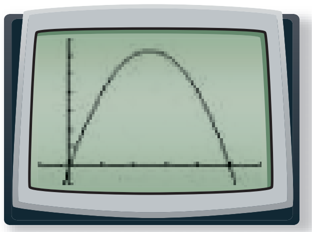
Producing a Graph

Enter Rule



Set Viewing Window

Display Graph



Use the software or calculator you have to produce and trace a graph for $I = p(50 - p)$ and estimate answers to the following questions. In each case, report your results with a sketch that shows how the answer is displayed on the calculator or computer screen.

a. What income is expected if the price is set at \$17?

b. What price(s) will lead to a daily income of about \$550?

c. How does the predicted income change as the price increases from \$0 to \$50?

d. What price will lead to maximum daily income from the bungee jump attraction?

3 **Using Computer Algebra Systems** When you can express the connection between two variables with a symbolic rule, many important questions can be written as equations to be solved. For example, to find the price per bungee jump that will give daily income of \$500, you have to solve the equation

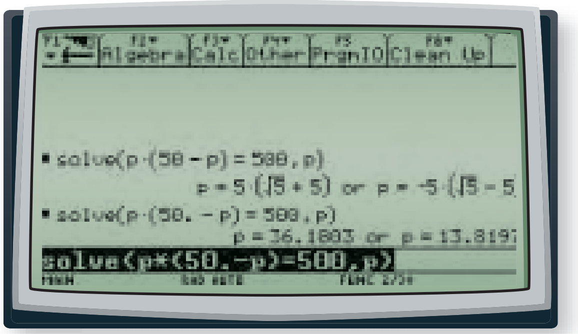
$$p(50 - p) = 500.$$

As you've seen, it is possible to estimate values of p satisfying this equation by scanning values in a table or tracing points on the graph of $I = p(50 - p)$. Computers and calculators are often programmed with computer algebra systems that solve automatically and exactly. One common form of the required instructions looks like this:

solve(p•(50–p)=500,p)

When you execute the command (often by simply pressing ENTER), you will see the solution(s) displayed in a form similar to that shown below.

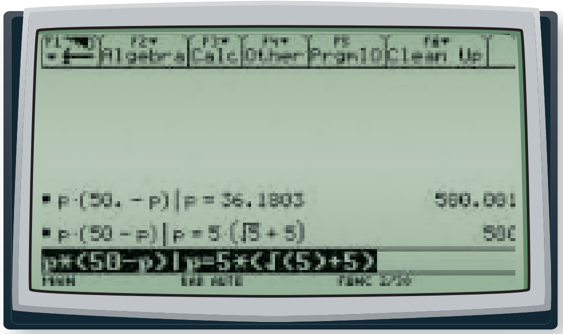
Solving an Equation



This display shows a special feature of computer algebra systems—they can operate in both *approximate* mode like graphing calculators or *exact* mode. (When some calculators with computer algebra systems are set in AUTO mode, they use exact form where possible. But they use approximate mode when an entry contains a decimal point.)

You can check both solutions with commands that substitute the values for p in the expression $p(50 - p)$. The screen will look something like the following display.

Evaluating an Expression



Computer algebra systems can do many other algebraic operations that you will learn about in future study. To get started, modify the instructions illustrated above to answer the following questions. In each case, check your results by using the same computer algebra system, scans of graphing calculator tables or traces of graphs, or arithmetic calculations.

a. What bungee jump price will give a predicted daily income of \$450? An income of \$0?

b. What daily income is predicted for a jump price of \$23? For a jump price of \$42?

c. What question will be answered by solving the equation $p(50 - p) = 225$? What is the answer?

d. How could you solve the equation $p(50 - p) = 0$ just by thinking about the question, "What values of p will make the expression $p(50 - p)$ equal to zero?"

Summarize the Mathematics

In this investigation, you developed skill in use of calculator or computer tools to study relations between variables. You learned how to construct tables and graphs of pairs of values and how to use a computer algebra system to solve equations.

a Suppose that you were given the algebraic rule $y = 5x + \frac{10}{x}$ relating two variables. How could you use that rule to find:

- the value of y when $x = 4$
- the value(s) of x that give $y = 15$

i. using a table of (x, y) values?
ii. using a graph of (x, y) values?
iii. using a computer algebra system?

b What seem to be the strengths and limitations of each tool—table, graph, and computer algebra system—in answering questions about related variables? What do these tools offer that makes problem solving easier than it would be without them?

Be prepared to share your thinking with the class.

Check Your Understanding

Weekly profit at the Starlight Cinema theater depends on the number of theater customers according to the rule $P = 6.5n - 2{,}500$. Use table, graph, and computer algebra system methods to complete Parts a–c. For each question:

- Report the setups you use to answer the questions by making and studying tables of (n, P) values. In each case, give the starting value and step size for the table that shows a satisfactory estimate of the answer.
- Report the window setups used to answer the questions by tracing a graph of the rule $P = 6.5n - 2{,}500$.
- Report the computer algebra system commands used to answer the questions and the results in approximate and exact modes.

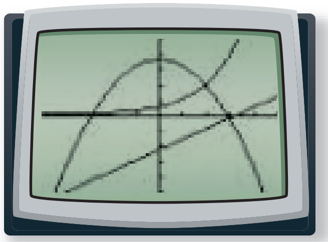
a. To find the **break-even point** for the business, you need to find the value of n that produces a value of P equal to 0. That means you have to solve the equation $0 = 6.5n - 2{,}500$. What values of n satisfy that equation?

b. What profit is predicted if the theater has 750 customers in a week?

c. What number of customers will be needed to make a profit of $1,000 in a week?

d. How could you answer the questions in Parts a–c if the only "tool" you had was your own arithmetic skills or a calculator that could only do the basic operations of arithmetic $(+, -, \times, \div)$?

Investigation 3 The Shapes of Algebra



The patterns you discovered while working on problems of earlier investigations illustrate only a few of the many ways that tables, graphs, and algebraic rules are useful in studying relations among variables. To find and use rules that relate independent and dependent variables or that predict change in one variable over time, it helps to be familiar with the table and graph patterns associated with various symbolic forms.

Getty Images

As you work on the explorations of this investigation, look for answers to this question:

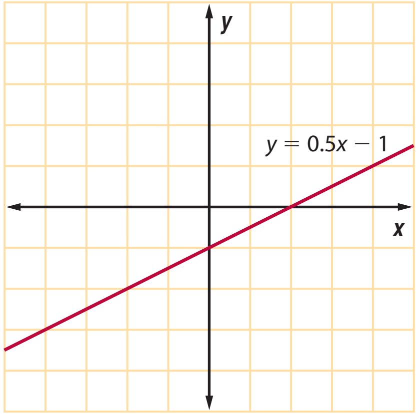
How do the forms of algebraic rules give useful information about the patterns in tables and graphs produced by those rules?

You can get ideas about connections between symbolic rules and table and graph patterns by exploration with a graphing calculator or a computer tool. You might find it efficient to share the following explorations among groups in your class and share examples within an exploration among individuals in a group.

In each exploration, you are given several symbolic rules to compare and contrast. To discover similarities and differences among the examples of each exploration:

- For each rule in the set, produce a table of (x, y) values with integer values of x and graphs of (x, y) values for x between –5 and 5.
- Record the table patterns and sketches of the graphs in your notes as shown here for the example $y = 0.5x - 1$.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$y = 0.5x - 1$	−3.5	−3.0	...								



- Then compare the tables, graphs, and symbolic rules in the exploration. Note similarities, differences, and connections between the symbolic rules and the table and graph patterns. Explore some other similar rules to test your ideas.
- Try to explain why the observed connection between rules and table/graph patterns makes sense.

Exploration 1. Compare the patterns of change in tables and graphs for these rules.

a. $y = 2x - 4$ **b.** $y = -0.5x + 2$ **c.** $y = 0.5x + 2$ **d.** $y = \frac{2}{x} - 4$

Exploration 2. Compare the patterns of change in tables and graphs for these rules.

a. $y = x^2$ **b.** $y = 2^x$ **c.** $y = -x^2$ **d.** $y = -x^2 + 2$

Exploration 3. Compare the patterns of change in tables and graphs for these rules.

a. $y = \frac{1}{x}$ **b.** $y = \frac{x}{3}$ **c.** $y = \frac{3}{x}$ **d.** $y = -\frac{5}{x}$

Exploration 4. Compare the patterns of change in tables and graphs for these rules.

a. $y = 3^x$ **b.** $y = x^3$ **c.** $y = 1.5^x$ **d.** $y = 4^x$

Summarize the Mathematics

As a result of the explorations, you probably have some ideas about the patterns in tables of (*x*, *y*) values and the shapes of graphs that can be expected for various symbolic rules. Summarize your conjectures in statements like these:

a If we see a rule like … , we expect to get a table like … .

b If we see a rule like … , we expect to get a graph like … .

c If we see a graph pattern like … , we expect to get a table like … .

Be prepared to share your ideas with others in your class.

Check Your Understanding

Each item here gives three algebraic rules—one of which will have quite different table and graph patterns than the other two. In each case, spot the "alien" rule and explain how and why its graph and/or table pattern will look different from the other two.

a. $y = \frac{10}{x}$
$y = 10x$
$y = x + 10$

b. $y = x^2 + 1$
$y = x + 1$
$y = 1 - x^2$

c. $y = 1.5x - 4$
$y = (1.5^x) - 4$
$y = 2^x$

d. $y = 1.5x - 4$
$y = 0.5x - 4$
$y = -1.5x - 4$

On Your Own

Applications

1. Members of the LaPorte High School football team have decided to hold a one-day car wash to raise money for trophies and helmet decals. They plan to charge $7.50 per car, but they need to pay $55 for water and cleaning supplies. Write a rule that shows how car wash profit is related to the number of car wash customers.

2. Juan and Tiffany work for their town's park department cutting grass in the summer. They can usually cut an acre of grass in about 2 hours. They have to allow 30 minutes for round-trip travel time from the department equipment shop to a job and back. What rule tells the time required by any job as a function of the number of acres of grass to be cut on that job?

3. When a summer thunder-and-lightning storm is within several miles of your home, you will see the lightning and then hear the thunder produced by that lightning. The lightning travels 300,000 kilometers per second, but the sound of the thunder travels only 330 meters per second. That means that the lightning arrives almost instantly, while the thunder takes measurable time to travel from where the lightning strikes to where you are when you hear it.

   You can estimate your distance from a storm center by counting the time between seeing the lightning and hearing the thunder. What formula calculates your distance from the lightning strike as a function of the time gap between lightning and thunder arrival?

4. Rush Computer Repair makes service calls to solve computer problems. They charge $40 for technician travel to the work site and $55 per hour for time spent working on the problem itself. What symbolic rule shows how the cost of a computer repair depends on actual time required to solve the problem?

5. The freshman class officers at Interlake High School ordered 1,200 fruit bars to sell as a fund-raising project. They paid $0.30 per bar at the time the order was placed. They plan to sell the fruit bars at school games and concerts for $0.75 apiece. No returns of unsold bars are possible. What rule shows how project profit depends on the number of bars sold?

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6. Janitorial assistants at Woodward Mall start out earning \$6 per hour. However, the \$75 cost of uniforms is deducted from the pay that they earn.

   a. Explain how the rule $E = 6.00h - 75$ shows how a new employee's earnings depend on the number of hours worked.

   b. How many hours will a new employee have to work before receiving a paycheck for some positive amount?

   c. How many hours will a new employee have to work to earn pay of \$100 before taxes and other withholdings?

   d. Sketch a graph of the rule relating pay earned to hours worked, and label points with coordinates that provide answers to Parts b and c.

7. Experiments with a bungee jump suggested the rule $L = 30 + 0.2w$ relating stretched length of the cord (in feet) to weight of the jumper (in pounds).

   a. What will be the stretched cord length for a jumper weighing 140 pounds?

   b. What jumper weights will stretch the cord to a length of at most 65 feet?

   c. Sketch a graph of the cord length relationship and label points with coordinates that give answers to Parts a and b.

   d. Study entries in a table of (w, L) values for $w = 0$ to $w = 300$ in steps of 10. Try to figure out what the values 30 and 0.2 tell about the bungee jump experience.

8. When promoters of a special Bruce Springsteen Labor Day concert did some market research, they came up with a rule $N = 15{,}000 - 75p$ relating number of tickets that would be sold to the ticket price.

   a. Income from ticket sales is found by multiplying the number of tickets sold by the price of each ticket. The rule $I = p(15{,}000 - 75p)$ shows how *income* depends on *ticket price*.

      i. What do the terms p and $(15{,}000 - 75p)$ each tell about how ticket price affects the concert business?

      ii. Why does the product give income as a function of ticket price?

   b. What ticket price(s) is likely to produce concert income of at least \$550,000?

   c. What is the predicted concert income if the ticket price is set at \$30?

   d. What ticket price is likely to lead to the greatest concert income?

   e. What ticket price(s) will lead to 0 income?

   f. Sketch a graph of the relationship between concert income and ticket price. Then label the points with coordinates that provide answers to Parts b, c, and d.

9 When members of the LaPorte High School football team ran their fund-raising car wash, they expected profit to be related to number of cars washed by $P = 7.50n - 55$.

a. If their goal was to earn a $500 profit, how many cars would the team have to wash?

b. How many cars would the team need to wash to break even?

10 Without use of your graphing calculator or computer software, sketch graphs you would expect from these rules. Explain your reasoning in each case.

a. $y = 7x^2 + 4$

b. $y = 7 - \frac{1}{4}x$

c. $y = 4^x - 7$

11 Without use of your graphing calculator or computer software, match the following four rule types to the tables below. Explain your reasoning in each case.

a. $y = ax + b$ **b.** $y = ax^2 + b$ **c.** $y = \frac{a}{x}$ **d.** $y = a^x$

I

x	−4	−3	−2	−1	0	1	2	3	4	5
y	18	11	6	3	2	3	6	11	18	27

II

x	−4	−3	−2	−1	0	1	2	3	4	5
y	16	14	12	10	8	6	4	2	0	−2

III

x	−4	−3	−2	−1	0	1	2	3	4	5
y	0.0625	0.125	0.25	0.5	1	2	4	8	16	32

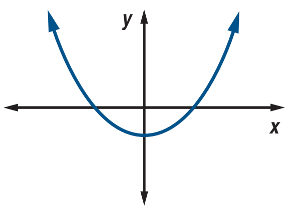
IV

x	−4	−3	−2	−1	0	1	2	3	4	5
y	−1.5	−2	−3	−6	error	6	3	2	1.5	1.2

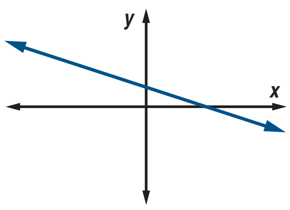
12 Without use of your graphing calculator or computer software, match the following four rule types to the graph sketches below. Explain your reasoning in each case.

a. $y = ax + b$ **b.** $y = ax^2 + b$ **c.** $y = \frac{a}{x}$ **d.** $y = a^x$

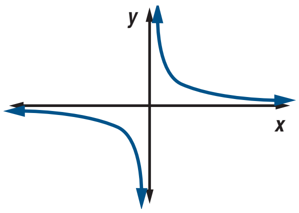
I



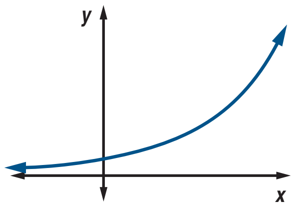
II



III



IV

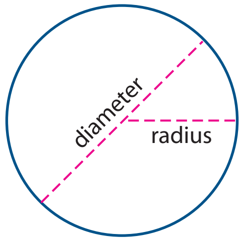


Connections

13 Three familiar formulas relate circumference and area of any circle to the radius or diameter of the circle. All three involve the number π, which is approximately 3.14.

Circumference: $C = \pi d$ and $C = 2\pi r$

Area: $A = \pi r^2$



a. Complete a table like the one below to show the pattern of change in circumference and area of a circle as the radius increases.

Radius r	0	1	2	3	4	5	10	20
Circumference C								
Area A								

b. Compare the pattern of change in area to the pattern of change in circumference as radius increases. Explain differences in the patterns of change by comparing the formulas.

c. How will the area change if the radius is doubled? If it is tripled?

d. How will the circumference change if the radius is doubled? If it is tripled?

e. Which change in the size of a circle will cause the greater increase in circumference—doubling the radius or doubling the diameter? Which of those changes will cause the greater increase in area?

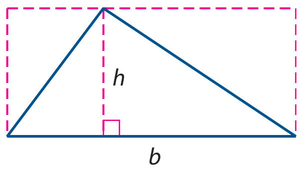
f. Tony's Pizza Place advertises 2-item, 10-inch pizzas for $7.95 and 2-item, 12-inch pizzas for $9.95. Which pizza is the better buy?

14 For polygons like triangles and rectangles, the formulas for perimeter and area often involve two variables—usually *base* and *height.*

Triangle Area: $A = \frac{1}{2}bh$

Rectangle Area: $A = bh$

Rectangle Perimeter: $P = 2b + 2h$



a. Complete entries in a table like the following to give a sample of triangle areas for different base and height values. The base values are in column A; the height values are in row 1; the areas go in cell B2 through J10. Then use the table patterns to answer questions in Parts b and c.

You might find it helpful to write a spreadsheet program to do the calculations.

Triangle Areas.xls

◇	A	B	C	D	E	F	G	H	I	J
1		1	2	3	4	5	6	7	8	9
2	1	0.5	1							
3	2	1	2							
4	3	1.5								
5	4									
6	5									
7	6									
8	7									
9	8									
10	9									
11										

b. Which change in the size of a triangle causes the greater increase in area—doubling the base or doubling the height?

c. How will the area of a triangle change if both the base and the height are doubled? What if both are tripled?

15 Create a table like that in Connections Task 14 to explore patterns of change in area of rectangles for base and height values from 1 to 9. Answer the same questions about the effects of doubling base and height for a rectangle.

16 Create a table like that in Connections Task 14 to explore patterns of change in perimeter of rectangles for base and height values from 1 to 9. Answer similar questions about the effects on perimeter of doubling base and height for a rectangle.

17 To answer Connections Task 14 with a spreadsheet, Mr. Conklin wrote some formulas for a few cells and then used "fill down" and "fill right" to get the rest of the sheet. Check his ideas in Parts a–c and explain why each is correct or not.

a. In cell B1, he entered "1". Then in cell C1, he wrote "=B1+1" and did "fill right" to complete row 1 of the table.

b. In cell A2, he entered "1". Then in cell A3, he wrote "=A2+1" and did "fill down."

c. In cell B2, he wrote "=0.5*$A2*B$1" and then did "fill down" and "fill right."

18 How could the instructions in Connections Task 17 be modified to produce the tables for

a. rectangle area?

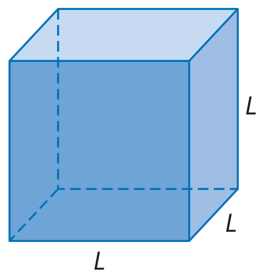
b. rectangle perimeter?

19 For each algebraic rule below, use your calculator or computer software to produce a table and then write a rule relating *NOW* and *NEXT* values of the y variable.

a. $y = 5x + 2$ **b.** $y = 10x - 3$

c. $y = 2^x$ **d.** $y = 4^x$

20 A cube is a three-dimensional shape with square faces.



a. If the length of an edge of a cube is L, write an expression for the area of one of its faces.

b. Write a rule that gives the total surface area A of a cube as a function of the length L of an edge.

c. Suppose you wished to design a cube with surface area of 1,000 square centimeters. To the nearest 0.1 centimeter, what should be the length of the edge of the cube?

Reflections

21 If you are asked to write a rule or formula relating variables in a problem, how would you decide:

a. what the variables are?

b. which of the variables seems most natural to be considered the independent variable and which the dependent variable?

c. what symbols should be used as shorthand names for the variables?

d. whether to express the relationship with "$y = \ldots$" or *NOW-NEXT* form?

22 If you enter the rule $y = 5x + 100$ in your calculator and press the GRAPH key, you might at first find no part of the graph on your screen. The plotted points may not appear in your graphing window. Talk with others in your class about strategies for making good window choices. Write down good ideas as a reminder to yourself and as a help to others.

23 Look back at your work for Part c of Connections Task 20.

a. What technology tool, if any, did you use in answering that question? How did you decide to use that tool?

b. How could you answer Part c using only the arithmetic capabilities of your calculator?

24 Suppose that you were asked to answer the following questions about a relationship between variables given by $y = 3.4x + 5$. Explain the tool *you* would choose for answering each question—calculation in your head, arithmetic with a calculator, study of a calculator- or computer-produced table of (x, y) values, study of a calculator- or computer-produced graph, or use of a computer algebra system command. Also, explain how you would use the tool.

a. Do the values of y increase or decrease as values of x increase?

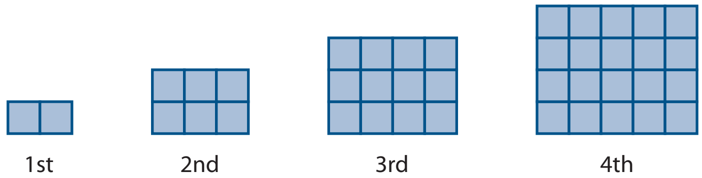
b. How rapidly do the values of y change as the values of x increase?

c. What is the value of y when $x = 7.5$?

d. What is the value of x when $y = 23.8$?

Extensions

25 The following sketches show the first four stages in a geometric pattern of rectangular grids made up of unit squares.



a. Describe geometrically how the grids change from one stage to the next.

b. What is the perimeter of the 5th rectangle?

c. What is the perimeter of the nth rectangle?

d. What is the area of the 5th rectangle?

e. What is the area of the nth rectangle?

26 Two different civic groups operate concession stands during games at the local minor-league baseball stadium. Group A sells hot dogs and soft drinks. Their profit P_A depends on the number of customers m and is given by $P_A = 3m - 100$. Group B sells ice cream. Their profit P_B depends on the number of customers n and is given by $P_B = 2n - 40$.

a. What are the break-even numbers of customers for each concession stand?

b. Is there any number of customers for which both stands make the same profit?

c. Which stand is likely to make the greater profit when the game draws a small crowd? When the game draws a large crowd?

27 Metro Cab Company charges a base price of \$1.50 plus 80¢ per mile. A competitor, Tack See Inc., charges a base price of \$2.50 plus 60¢ per mile.

a. What rules give the charge for a trip with each company as a function of the length of the trip?

b. If you need to travel 3 miles, which cab company is the least expensive?

c. If you need to travel 15 miles, which cab company is the least expensive?

d. For what trip length are the costs the same for the two cab companies?

28 Suppose, as part of an agreement with her father to do some work for him during the summer, Tanya will receive 2¢ for the first day of work, but every day after that her pay will double.

a. What rule shows how to calculate Tanya's daily pay p on work day n.

b. What rule using *NOW* and *NEXT* shows how Tanya's pay grows as each additional day of work passes?

c. If Tanya's pay for a day is \$10.24, how many days has she worked?

d. Find Tanya's pay for a day after she has worked 20 days.

e. For how many days will she earn less than \$20 per day?

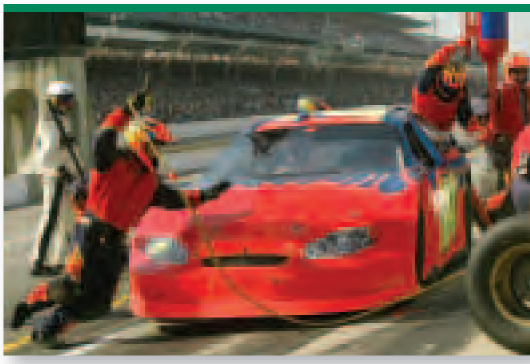
29 One car rental company charges \$35 per day, gives 100 free miles per day, and then charges 35¢ per mile for any miles beyond the first 100 miles per day.

a. What rule gives the charge for renting for one day from this company as a function of the number of miles m driven that day?

b. What rule gives the charge for renting from this company as a function of both miles driven m and number of days d?

c. A business person plans a trip of 300 miles that could be made in one day. However, she would arrive home late and is considering keeping the rental car until the next morning. What would you suggest? Explain your reasoning.

30 At the start of a match race for two late-model stock cars, one stalls and has to be pushed to the pits for repairs. The other car roars off at an average speed of 2.5 miles per minute. After 5 minutes of repair work, the second car hits the track and maintains an average speed of 2.8 miles per minute.



a. How far apart in the race are the two cars 5 minutes after the start? How far apart are they 10 minutes after the start?

b. What three rules can be used to calculate the distance traveled by each car and the distance between the two cars at any time after the start of the race?

c. On the same coordinate axes, make graphs displaying the distances traveled by each car as a function of time. Use a horizontal scale that allows you to see the first 60 minutes of the race.

d. On a different set of axes, make another graph showing the distance between the two cars as a function of time since the start of the race.

e. Explain what the patterns of the graphs in Parts c and d show about the progress of the race.

f. Write and solve equations that will answer each of these questions about the race:

i. How long after the start of the race will it take the first car to travel 75 miles? The second car?

ii. At what time after the start of the race will the second car catch up to the first car?

Brent Smith/Reuters/CORBIS

Review

31 Evaluate each expression if $x = 1$, $y = 3$, $a = -1$, and $b = 2$.

a. $a^2x + b^3y$

b. $a^2(x + by)$

c. $\dfrac{x^3(y + 1)}{by + a^3}$

32 A random sample of 100 students is chosen to survey about lunch preferences.

25 say their first choice is pizza.

30 say their first choice is chicken nuggets.

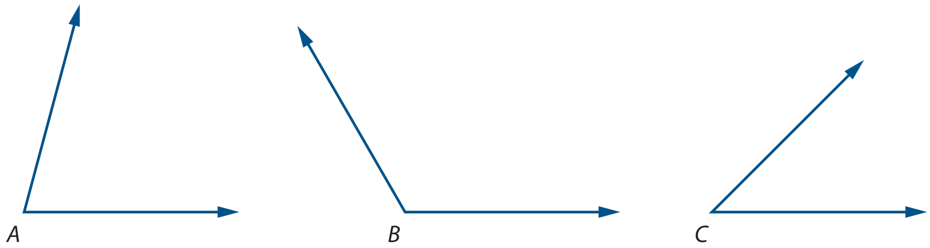
15 say their first choice is salad bar.

20 say their first choice is tacos.

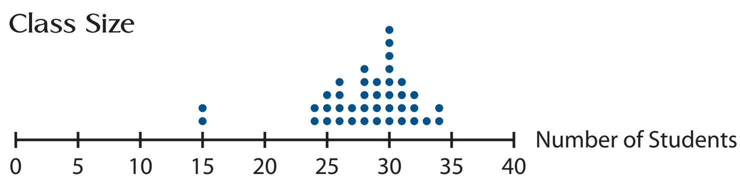
10 say their first choice is subs.

If the entire school population is 1,500 students, how many students can you predict will have pizza as a first choice?

33 Estimate the measure of each angle. Check your estimates with a protractor.



34 The dot plot below indicates the number of students in the 40 first-hour classes at Lincoln High School.



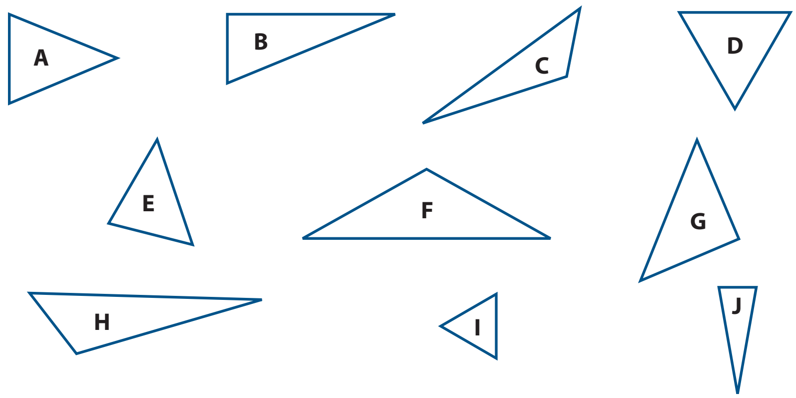
a. What was the smallest class size?

b. What was the largest class size?

c. What percent of the classes had 30 students in them?

d. What percent of the students had fewer than 25 students in them?

35 Consider the triangles drawn below. Assume that angles that look like right angles are right angles and that segments that appear to be the same length are the same length.



a. Identify all acute triangles.

b. Identify all obtuse triangles.

c. Identify all isosceles triangles.

d. Identify all scalene triangles.

e. Identify all equilateral triangles.

f. Identify all right triangles.

36 The lengths of the sides of a triangle are 4, 5, and 6 inches. These sides are scaled up by multiplying by a factor so that the length of the longest side of the new triangle is 10 inches.

a. What is the scale factor?

b. What are the lengths of the two shorter sides of the new triangle?

LESSON 4

Looking Back

In your work on problems and explorations of this unit, you studied many different patterns of change in variables. In some cases, the aim was to describe and predict patterns of change in a dependent variable that are caused by change in values of an independent variable. In other cases, the goal was to describe and predict patterns of change in values of a single variable with the passage of time. For example:

For each weight attached to a bungee cord, there was a predicted stretched length for the cord.

For each year after the census in 2001, there was a predicted population of Alaskan bowhead whales.

For each possible price for a bungee jump at Five Star Amusement Park, there was a predicted daily income from the attraction.

In mathematics, relations like these—where each possible value of one variable is associated with exactly one value of another variable—are called **functions**. The use of the word "function" comes from the common English phrase that appears in statements like "cord stretch *is a function of* attached weight" or "average speed for the Iditarod Sled Race *is a function of* time to complete the race."

Many functions of interest to mathematicians have no particular cause-and-effect or change-over-time story attached. The only condition required for a relationship to be called a function is that each possible value of the independent variable is paired with one value of the dependent variable.

As a result of your work on Lessons 1–3, you should be better able to:

- recognize situations in which variables are related in predictable ways,
- use data tables and graphs to display patterns in those relationships,
- use symbolic rules to describe and reason about functions, and
- use spreadsheets, computer algebra systems, and graphing calculators to answer questions about functions.

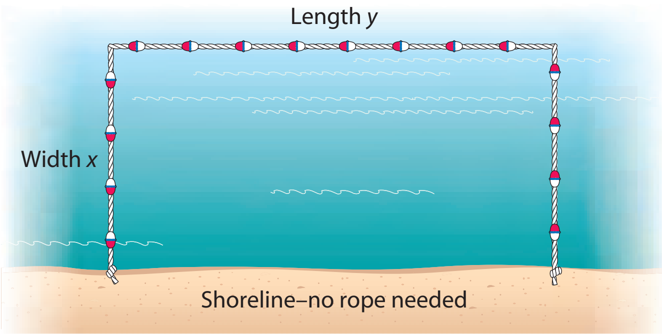
The tasks in this final lesson of the *Patterns of Change* unit will help you review, pull together, and apply your new knowledge as you work to solve several new problems.

Anne-Marie Weber/Taxi/Getty Images

1 **Five Star Swimming** In addition to bungee jumping and rides like roller coasters and a Ferris wheel, Five Star Amusement Park has a large lake with a swimming beach and picnic tables.

Every spring when the park is preparing to open, lifeguards at the beach put out a rope with buoys outlining the swimming space in the lake. They have 1,000 feet of rope, and they generally outline a rectangular swimming space like that shown below.

When working on this task one year, the lifeguards wondered whether there was a way to choose dimensions of the rectangular swimming space that would provide maximum area for swimmers.



a. Complete entries in a table like this, showing how dimensions of the swimming space are related to each other. Then write a rule giving y as a function of x.

Width *x* (in feet)	50	100	150	200	250	300	350	400	450
Length *y* (in feet)									

b. One of the lifeguards claimed that the rule in Part a can be used to write another rule that shows how area A of the swimming space depends on choice of the width x. She said that $A = x(1{,}000 - 2x)$ would do the job. Is she right? How do you think she arrived at this area rule?

c. Use the area function in Part b and strategies you have for reasoning about such relationships to answer the following questions. To show what you've learned about using different tools for studying functions:

- Answer one question by producing and scanning entries in a table of values for the area function.
- Answer another question by producing and tracing coordinates of points on a graph of the function.
- Then answer another question using a computer algebra system equation solver.

When you report your results, explain your strategies as well.

i. What dimensions of the swimming space will give maximum area? What is that area?

ii. What dimensions will give a swimming area of 100,000 square feet?

iii. What dimensions will give a swimming area of 50,000 square feet?

2 **Borrowing to Expand** When the Five Star Amusement Park owners decided to expand park attractions by adding a new giant roller coaster, they borrowed $600,000 from a local bank. Terms of the bank loan said that each month interest of 0.5% would be added to the outstanding balance, and the park would have to make monthly payments of $10,000. For example, at the end of the first month of the loan period, the park would owe $600{,}000 + 0.005(600{,}000) - 10{,}000 = \$593{,}000$.

a. Make a table showing what the park owes the bank at the end of each of the first 12 months.

b. Write a *NOW-NEXT* rule that shows how the loan balance changes from one month to the next.

c. Use a calculator or spreadsheet strategy to find out how long it will take to pay off the loan.

d. Plot a graph showing the amount owed on the loan at the end of months 0, 6, 12, 18, … until it is paid off. Describe the pattern of change in loan balance over that time.

3 **Setting the Price** Because Five Star managers expected the new roller coaster to be a big attraction, they planned to set a high price for riders. They were unsure about just what that price should be. They decided to do some market research to get data about the relationship between *price per ride* and *number of riders* that would be expected each day.

a. Complete a table that shows how you believe the *number of riders* will depend on the *price per ride.* Explain the pattern of entries you make and your reasons for choosing that pattern.

Price per Ride (in dollars)	0	5	10	15	20	25	30	35
Number of Riders								

b. Add a third row to the table in Part a to give the predicted *income* from the new roller coaster for each possible *price per ride.* Then plot the (*price per ride, income*) data and describe the pattern of change relating those variables.

c. Estimate the price per ride that will give maximum daily income.

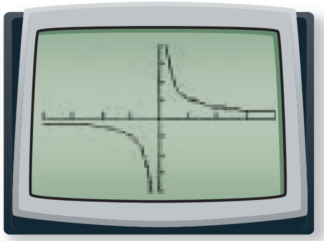
d. What factors other than price are likely to affect daily income from the roller coaster ride? How do you think each factor will affect income?

Alan Oliver/Alamy

4. Without using a graphing calculator or doing any calculation of (x, y) values, match each of the following functions with the graph that best represents it.

a. $y = -0.5x - 4$ **b.** $y = x^2 - 4$ **c.** $y = \frac{4}{x}$ **d.** $y = (1.5^x)$

I



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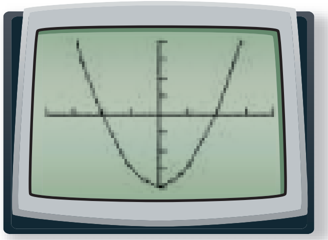

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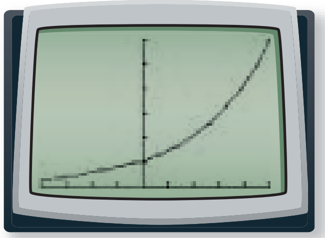

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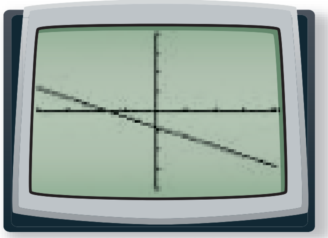

Summarize the Mathematics

When two variables change in relation to each other, the pattern of change often fits one of several common forms. These patterns can be recognized in tables and graphs of (x, y) data, in the rules that show how to calculate values of one variable from given values of the other, and in the conditions of problem situations.

a Sketch at least four graphs showing different patterns relating change in two variables or change in one variable over time. For each graph, write a brief explanation of the pattern shown in the graph and describe a problem situation that involves the pattern.

b Suppose that you develop or discover a rule that shows how a variable y is a function of another variable x. Describe the different strategies you could use to:

- **i.** Find the value of y associated with a specific given value of x.
- **ii.** Find the value of x that gives a specific target value of y.
- **iii.** Describe the way that the value of y changes as the value of x increases or decreases.
- **iv.** Find values of x that give maximum or minimum values of y.

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

Write, in outline form, a summary of the important mathematical concepts and methods developed in this unit. Organize your summary so that it can be used as a quick reference in future units and courses.